A microfounded model for fiscal policy analysis*

PRELIMINARY DRAFT

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1 Introduction

The purpose of this document is to introduce the reader to a microfounded model of the Norwegian economy designed for fiscal policy analysis, and to present some preliminary model results and simulations. At the outset is should be stressed that the model presented here is work in progress and should thus be considered only a basis for discussion and further extensions.

The model presented in the following belongs to the class of small open economy DSGE (dynamic stochastic general equilibrium) models. The model describes the dynamic adjustment of a a small open economy to various macroeconomic shocks and fiscal policy changes (focusing particularly on the short- to medium-term) while at each point in time ensuring that demand and supply in all modeled markets are in equilibrium. The model features two types of households, (i) rule-of-thumb households that consume all of their income in each period and, thus, are not forward-looking and (ii) optimizing households that choose consumption and investment and set wages intertemporally through utility maximization. Firms in the economy fulfill various tasks, including the production of goods using capital and labor provided by households, the import and export of goods from and to foreign markets and the combination of domestically produced and imported goods to final consumption and investment goods. The government levies a variety of distortionary taxes on firms and households and, particularly important in the Norwegian context, uses withdrawals from public savings to finance its expenditures, consisting of public investments, purchases of goods and services, public employment wages as well as transfers to households. The central bank sets the nominal interest rate aiming for price and output stability. Section 2 will introduce the model in more detail, first in a non-technical summary, later and only for the interested reader using mathematical equations supported by derivations in the appendix.

The model has been parameterized to roughly reflect the Norwegian mainland economy. In particular, we have calibrated the implied steady state of the model economy to a number of long-run moments in the data, including consumption-, investment-, export and import-to-GDP ratios, labor market characteristics involving the (un-)employment and labor force participation rate as well as a range of government related variables including the level of

*The views and conclusions expressed in this paper are the responsibility of the authors alone and should not be interpreted as reflecting the views of the Norwegian Ministry of Finance.

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expenditure and revenue components described above. At this point the dynamic properties of the model are determined by qualitatively (and to a limited extent quantitatively) matching impulse responses of macroeconomic shocks to comparable DSGE models from policy institutions and academia. A more rigorous and data-driven approach to the determination of the dynamic parameters of the model, as well as a systematic comparison with related macroeconomic models of Norway, such as KVARTS and NEMO, will follow as an extension. Section 3 provides more information on the calibration procedure as well as the fit of the steady state.

Section 4 of this report illustrates the preliminary results of the current model setup including stochastic simulations to aggregate shocks and deterministic simulations that allow the analysis of long-run shifts in policy variables. The simulation results will be discussed with the intention of demonstrating possible usage scenarios for the final model. Finally, section 5 provides a summary and an overview on the way ahead of the modeling project.

2 Model

2.1 Non-technical summary

Figure 1: Schematic overview of the model

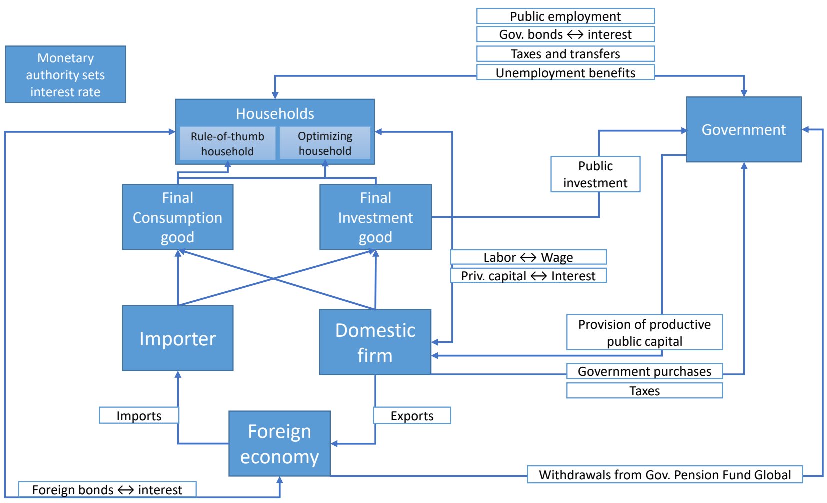


Figure 1 provides a graphical overview on the model. In the following we will provide a non-technical summary of the different model components shown in the figure and their interactions. While the model set-up is quite general and in principle applicable to different economies, certain model characteristics reflect particularities of the Norwegian economy.

Small open economy The model consists of a foreign economy block capturing foreign output, interest rate, inflation and the oil price. While these four variables are connected among each other the domestic economy cannot influence them. This captures the simplifying assumption that the Norwegian economy is sufficiently small to not affect the macroeconomic dynamics of the world economy in a quantitatively important way.

Households There are two types of household in the economy. The rule-of-thumb (RoT) household's expenditure only consist of the the final consumption good. Its income consists of labor-income both from employment in domestic firms and the government as well as from public transfers and unemployment benefits. The RoT household is not forward-looking and simply consumes all of its income at each point in time. The optimizing household instead intertemporally allocates its consumption expenditures and thus uses investment goods and foreign or government bonds to store its wealth. Additionally to the same income sources as the RoT household the optimizing household generates income through interest on its bond holdings, by renting out its capital stock and through its ownership of domestic firms that yield dividends. Both households pay a range of different taxes, including a value-added tax on consumption, an income tax on labor as well as, and only in the case of the optimizing household, other income from capital, bonds and dividends.

Wages, employment and interest rates Wages in the economy are negotiated between the members of the optimizing household and domestic firms. Wage bargaining power in the optimizing household arise as its member are assumed to provide specialized labor that is not perfectly substitutable across specialization. However, wages cannot be set arbitrarily high as firms demand specialization-specific labor given their wage level. Wages payed by the government as well as wages earned by RoT household members are assumed to follow these negotiated wages (proportionally). Unemployment arises as wages are negotiated to be above their laissez-faire, i.e. market-clearing level. Moreover, members of the optimizing household also differ in their degree of disutility that they experience when working. Those with the highest degree of disutility might, depending on the prevailing employment opportunities and wages, chose to leave the labor-force altogether. The rental rate of capital is determined in the competitive equilibrium and neither chosen by firms nor households. The interest rate on domestic bonds is set by the central bank who aims to hold inflation and output stable.

Production Production is undertaken by domestic firms. To produce differentiated goods these use labor and a composite capital stock, consisting of public and private capital. Firms have monopolistic power on their differentiated goods, enabling them to set prices with a markup over marginal cost. The output is sold both in domestic and exporting markets at distinct home and export prices. Importers buy homogeneous foreign goods in a competitive market at world prices, reprocess and transform those homogeneous goods into either differentiated consumption or investment goods and sell in the domestic market with a market power to the final good sector while not using any domestic inputs to their production. The final good sector's role in the model is to bundle differentiated domestically produced products and imported goods to produce the final consumption and investment good.

Frictions Both, households and firms are subject to nominal and real frictions. Households incur adjustment costs when adjusting the level of investment as well as when renegotiating their nominal wages. Firms face adjustment costs when changing prices. In general these frictions improve the model's ability to account for the sluggishness with which economies react to aggregate shocks. In particular, price adjustment costs faced by importers and domestic firms for exported goods prevent sudden changes in the exchange rate to translate themselves one to one onto the respective selling prices.

Government Finally, the government levies a variety of taxes, including consumption, labor income, capital income and lump-sum taxes on households. Since this is a model that aims to capture the salient features of the Norwegian fiscal policy environment, we include in our model also withdrawals from a public foreign asset position. These public savings reflecting the Norwegian Government Pension Fund Global are, however, not explicitly modeled. Taxes and withdrawals are used to finance government expenditures, consisting of unemployment benefits, public purchases of goods and services, public employment and investments into public capital stock. The model allows the latter to only induce a demand-effect on the economy, or additionally to be productivity-enhancing by increasing the public capital stock employed in private production. The behavior of the government can either follow fiscal rules or exogenous processes such that model can be used to analyze economic consequences of policy changes.

We now continue with an in-depth presentation of the model.

2.2 Households

Following Mankiw [2000] and Galí et al. [2007], we assume two types of representative households in the model, namely an optimizing household, denoted with a o-superscript, and a rule-of-thumb household, denoted with a r-superscript. While optimizing households choose current consumption and set their wage with a view of maximizing their lifetime utility, rule-of-thumb individuals simply consume all available income net of taxes. Furthermore, in order to capture involuntary unemployment, we adopt the framework of Galí et al. [2012] and Stähler and Thomas [2012], in which the optimizing household consists of a continuum of individuals differing in the type of labor service they are specialized in and in their personal disutility of work. Optimizing households set wages in a competitive labor market subject to adjustment costs, while rule-of-thumb households follow the wages set by optimizing households. The share of rule-of-thumb households is denoted by $0 \leq \omega \leq 1$.

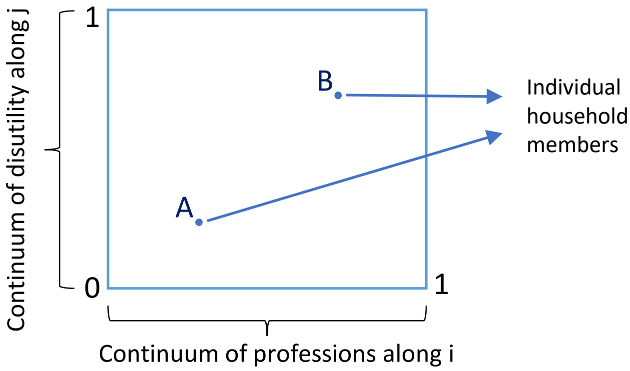
2.2.1 Optimizing households

Lifetime utility The members of the optimizing household are represented by the unit square $(i, j) \in [0, 1] \times [0, 1]$, indexing the members according to their type of labor service they are specialized in (indexed by i) and the degree of disutility of work (indexed by j). Figure 2 illustrates this household structure. The full square represents the optimizing household, which consists of infinitely many points representing the individual household members. An individual within the household can be identified by its coordinates within the square which represent on the horizontal axis its profession and on the vertical axis its degree of disutility of work. For example, household member A works in a different profession than household member B who in turn exhibits a higher degree of labor disutility than member A. As we will see later, the top part of the square, that exhibits a relatively high degree of disutility of labor, will choose to stay out of the labor force. The labor disutility of members of type (j) is assumed to be given by $\theta \cdot (j^{\psi})$ if employed and zero otherwise where θ is the weight of disutility of labor. The parameter ψ affects the implied Frisch-elasticity of the model.[1]

The preferences of the household are additively separable in consumption, C_t^o, labor disutility and utility-providing public goods, G_t^u. We can thus express expected lifetime utility of the optimizing household at time s, denoted by $U_s(j)$, by

[1] As Stähler and Thomas [2012] note, ψ is not simply the inverse of the Frisch elasticity because of the existence of unemployment, public employment and unemployment benefits.

Figure 2: Representation of the optimizing household



$$
\begin{aligned}
U_s &= E_s \sum_{t=s}^{\infty} \beta^t \epsilon_{g,t} \left[\frac{(C_t^o - H_t)^{1-\sigma}}{(1-\sigma)(1-h)^{-\sigma}} - \theta \int_0^1 \int_0^{N_t^o(i)} j^{\psi} dj di + \theta_G \frac{(G_t^u)^{1-\sigma_G}}{1-\sigma_G} \right] \\
&= E_s \sum_{t=s}^{\infty} \beta^t \epsilon_{g,t} \left[\frac{(C_t^o - H_t)^{1-\sigma}}{(1-\sigma)(1-h)^{-\sigma}} - \theta \int_0^1 \frac{N_t^o(i)^{1+\psi}}{1+\psi} di + \theta_G \frac{(G_t^u)^{1-\sigma_G}}{1-\sigma_G} \right]. \qquad (1)
\end{aligned}
$$

Note, that we only consider consumption and public good consumption at the household level while the household's disutility of labor is given by the individual degree of disutility aggregated across all household members. For each profession, we only integrate up to the employment rate in profession i, given by $N_t^o(i)$, since unemployed household members do not incur any disutility of labor. The term $\epsilon_{g,t}$ represents a preference shock, modeled through an AR(1) process. As evident from above equation, utility on consumption also depends on the term $H_t = hC_{t-1}^o$ which denotes an external habit over average consumption.[2] The term $(1-h)^{-\sigma}$ ensures, that the first-order condition of consumption, which is derived below, does not depend on h in the steady state, making h are purely dynamic parameter. The intertemporal elasticity of substitution of consumption goods is captured by the parameter σ. Finally, G_t^u, are utility-providing government expenditures which are weighted against other components of life-time utility by the factor θ_G while the intertemporal substitutability is governed by σ_G. G_t^u consist of the sum of three components, namely government purchases $P_{h,t}C_t^G$, government capital depreciation $P_t^i \delta_{KG} K_t^P$ as well as the government wage bill $w_t^G N_t^G$ which are collectively a proxy for the goods and services produced by the government and available to all individuals in the economy. While these government expenditures are not wasteful and increase the utility of households, they do not affect the consumption-saving decision (nor wage-setting) since the corresponding term enters the utility function only additively.

For convenience, we provide an overview on variables introduced so far:

- i and j: household member indexes for profession and disutility, respectively
- C_t^o: consumption at time t of the overall household
- $H_t = hC_{t-1}^o$: external habit over average consumption

[2] Note, that the household does not take into account that its current consumption will affect the utility enjoyed from future consumption. Note also, that the habits as modeled here differ quite substantially from deep habits literature sparked by Ravn et al. [2006], where habits are formed on individual consumption goods, rather than on the aggregate composite consumption index.

- $N_t^o(i)$: employment rate in profession i
- $\epsilon_{g,t}$: preference shock
- G_t^u: utility-providing government expenditures

Budget constraint The representative household obtains income from supplying labor, renting out accumulated capital to firms, yields on foreign and domestic bonds and dividends from owned firms' profits. The sum of all income will be referred to as the ordinary income and is denoted by OI_t and given by

$$OI_t = LI_t + R_t^k u_t K_t^o + \frac{P_{t-1}}{P_t} D_{t-1}^o (R_{t-1} - 1) + e_t \frac{P_{t-1}^*}{P_t} B_{t-1}^o (R_{t-1}^* \phi_{t-1} - \frac{e_{t-1}}{e_t} \frac{P_t}{P_{t-1}}) + DIV_t^o \tag{2}$$

where LI_t denotes real labor income and will be defined below. The second term captures income from capital rental, with K_t^o denoting the capital stock and R_t^k the rental rate of capital. The third term represents interest income from last period's holdings of government bonds, D_{t-1}^o, which is translated into this period's value by dividing through the inflation rate $\pi_t = \frac{P_t}{P_{t-1}}$. The nominal gross risk-less interest rate is denoted by R_t and is set by the monetary authority, which will be discussed further below. The foreign gross interest rate is given by R_t^*. The second-to-last term captures interest income from foreign bonds while taking into account valuation effects from changes in the real exchange rate,[3] following Adolfson et al. [2013]. More specifically, e_t denotes the nominal exchange rate (i.e. the value of one unit of foreign bond in the domestic currency), P_t^* captures the foreign price level, B_{t-1}^o last period's holdings of foreign bonds.[4] Finally, the effective interest rate on foreign bonds is subject to a debt elastic interest rate premium ϕ_t. In doing so, we follow Adolfson et al. [2008] and set

$$\phi_t := \exp(-\chi_A(A_t - \overline{A}) - \chi_e(\frac{e_{t+1}}{e_t}\frac{e_t}{e_{t-1}} - 1) + \chi_O(OILR_t - \overline{OILR}) + \tilde{\phi}_t) \tag{3}$$

where $\tilde{\phi}_t$ is a risk premium shock and $A_t = \frac{Q_t B_t}{\overline{Y}}$. The interest rate premium implies, that for net-lenders to international debtors the interest rate on international bonds is lower than the risk-free interest rate ($\phi_t < 1$) and for net-borrower the interest rate is higher than risk-free rate ($\phi_t > 1$). Furthermore, the magnitude of increases in the nominal exchange rate and, in addition to Adolfson et al. [2008], withdrawals from public assets exceeding[5] the steady-state level additionally affect the risk premium.[6] The last term in equation (2), DIV_t^o, denote dividends that enter as income in the household's budget, since the household is assumed to own the firms and thus receive their profits. For later convenience we define at this point the real exchange rate as

$$Q_t := e_t P_t^* / P_t \tag{4}$$

and the change in nominal exchange rate as

[3]It holds, that $e_t \frac{P_{t-1}^*}{P_t} B_{t-1}^o (R_{t-1}^* \phi_{t-1} - \frac{e_{t-1}}{e_t} \frac{P_t}{P_{t-1}}) = e_t \frac{P_{t-1}^*}{P_t} B_{t-1}^o (R_{t-1}^* \phi_{t-1}) - e_{t-1} \frac{P_{t-1}^*}{P_{t-1}} B_{t-1}^o$. Hence the first term captures the whole nominal payoff of the bond, including principal. The second term after the minus ensures that the principal is deducted from taxation, thereby capturing any income or losses through foreign asset appreciation or depreciation, respectively.

[4]Note, that these foreign bonds will be calibrated to capture only foreign assets held by the private sector. Hence, public foreign assets are not on the household's balance sheet.

[5]We refer to these withdrawals as $OILR_t$ adapting the Notation from NEMO I.

[6]Alternatively, the risk premium can also be modeled as $\phi_t := \exp(-\chi_A(A_t - \overline{A}) + \tilde{\phi}_t)$ as in Schmitt-Grohé and Uribe [2003] or $\phi_t := \exp(-\chi_A(A_t - \overline{A}) - \chi_R(R_t^* - R_t - (\bar{R}^* - \bar{R})) + \tilde{\phi}_t)$ as in Christiano et al. [2011]. All three options are available in the model code.

$$(\delta e)_t := \frac{e_t}{e_{t-1}} = \frac{Q_t}{Q_{t-1}} \frac{\pi_t}{\pi_t^*}. \tag{5}$$

Returning to the definition of ordinary income above, we define labor income LI_t as

$$LI_t = \frac{1}{P_t} \left(\int_0^1 W_t(i) N_t^{o,P}(i) di + W_t^G N_t^{o,G} \right) \tag{6}$$

where $W_t(i)$ is the nominal wage rate in profession i and $N_t^{o,P}(i)$ the private-sector employment rate. The first term within the brackets thus captures the aggregated nominal income from private-sector employment across all household terms. The second term represents the household's income from public employment, where the nominal government wage is given by W_t^G and the public employment rate by $N_t^{o,G}$. We assume that government wages are proportional to private wages, i.e. $W_t^G = WG^m W_t$, where WG^m is a constant and W_t the average private-sector wage rate. Note, that the sum of private, $N_t^{o,P}(i)$ and public employment rates, $N_t^{o,G}$, is defined to be the total employment rate introduced in the utility function, namely $N_t^o(i)$. The model thus features not only private but also public employment and resembles the set-up in Stähler and Thomas [2012] and Gadatsch et al. [2016]. The public employment rate is set by the fiscal authority and will be introduced further below.

Total taxes T_t paid by the optimizing household consist of a lump-sump tax $T_t^{L,o}$, a flat tax on ordinary income τ_t^{OI}, an additional labor-income tax, the so-called bracket tax, τ_i^{BT}, and social security contributions that are applied to labor income $\tau_t^{SS,H}$ as well as an allowance on capital depreciation net of capital utilization. Total taxes are then given by

$$T_t = T_t^{L,o} + \tau_t^{OI} OI_t + (\tau_t^{BT} + \tau_t^{SS,H}) LI_t - \tau_t^{OI} (\delta P_t^i K_t^o + \gamma_t^u). \tag{7}$$

The parameter δ captures the depreciation rate of capital while γ_t^u denotes the costs / benefits of capital over-/under-utilization. Both aspects will be discussed in further detail below. We define $\tau_t^W = \tau_t^{OI} + \tau_t^{BT} + \tau_t^{SS,H}$ as the overall tax rate on labor income. The household's budget constraint (in nominal terms) is given by

$$\begin{aligned} &\underbrace{P_t C_t^o (1 + \tau_t^C)}_{\text{Consumption exp.}} + \underbrace{P_t^I I_t^o}_{\text{Investment exp.}} + \underbrace{P_t \gamma_t^o}_{\text{Adjustment costs}} \\ &+ \underbrace{P_t D_t^o - P_{t-1} D_{t-1}^o}_{\text{Increase in Holdings of Gov. bonds}} + \underbrace{e_t P_t^* B_t^o - e_t P_{t-1}^* B_{t-1}^o}_{\text{Increase in holdings of foreign bonds}} \\ = \; &P_t OI_t - P_t T_t + \underbrace{P_t UB_t \left(L_t^o - N_t^o\right)}_{\text{Unemployment benefits}} + \underbrace{TR_t^o}_{\text{Lump-sum transfers}} . \end{aligned} \tag{8}$$

Here, the left hand side of the budget constraint shows the expenditures of households whereas the right hand side presents the income of household net of taxes. Several terms warrant further explanation. The term τ_t^C captures the tax rate paid on consumption. Apart from consumption expenditures and savings in forms of increases in the holdings of domestic and foreign bonds, the household spends resources on investments, I_t^o, that are used to build up the capital stock as well as on adjustment costs. These capture two types of costs, namely $\gamma_t^o = \gamma_t^u + \gamma_t^W$ where γ_t^u will be discussed further below. The second term captures age adjustment costs, which ensures that it is costly for household members to renegotiate nominal wages and thus produces a sluggish and thus more realistic response of wages to various shocks in the economy. The functional form is adopted from Gadatsch et al. [2016] and is given by

$$\gamma_t^W = \int_0^1 \frac{\chi_W}{2} \left(\frac{W_t(i)/W_{t-1}(i)}{[(W_{t-1}/W_{t-2})^{\chi_{aW}}]\,\overline{\pi}^{(1-\chi_{aW})}} - 1 \right)^2 w_t di \tag{9}$$

with χ_{aW} being the weight on wage indexation and $w_t = W_t/P_t$ the real wage.[7]. On the income side of the budget constraint, we observe apart from the already defined ordinary income net of taxes a term capturing unemployment benefits UB_t that is payed out to the fraction of household members that have entered the labor force but are not in employment. As we will show later, this fraction is uniform across all professions such that $L_t^o(i) - N_t^o(i) = L_t^o - N_t^o$. Finally, TR_t^o captures lump-sum transfers to the optimizing household.
For convenience, we provide an overview on the main variables introduced in this subsection:

- P_t: domestic CPI; P_t^*: foreign CPI
- τ_t^C is the tax rate paid on consumption.
- I_t^o: investments, P_t^I: the price of investment, $P_t^i := P_t^I/P_t$: relative price of investment
- D_t^o: real value of domestic bonds of government purchased at time t ($>$0 if individual is lender to government)
- B_t^o: real value of international bonds purchased at time t ($>$0 if individual is lender)
- e_t: nominal exchange rate; $Q_t := e_t P_t^*/P_t$: real exchange rate
- $N_t^{o,P}(i)$: employment rate in the private sector of profession i; $N_t^{o,G}$: the employment rate in the government sector; total employment is given by $N_t^o(i) = N_t^{o,P}(i) + N_t^{o,G}$
- $L_t^o(i)$ denotes the profession-specific participation rate.
- R_t: risk-less interest rate, R_t^* foreign risk-less interest rate
- UB_t: unemployment benefits; TR_t^o: lump-sum transfer
- K_t^o: amount of capital at the beginning of period t (thus it is determined at $t-1$ and treated as K(-1) in Dynare)
- R_t^k: rental rate of capital
- DIV_t^o: dividends from holding shares in firms

Labor supply and demand Following Stähler and Thomas [2012] and Gadatsch et al. [2016] we assume that the household member (i,j) decides whether to participate in the labor market, taking into account the household welfare and its personal disutility of work given current labor market conditions. The individual j of profession i will find it optimal to participate in the labor market in period t if and only if

$$\lambda_t \left((1-\tau_t^W)(w_t(i)N_t^{o,P}(i) + w_t^G N_t^{o,G}) + UB_t(j - N_t^o(i)) \right) \geq N_t^o(i)\epsilon_{g,t}\theta j^{\psi}.$$

The left-hand side of the equation captures the utility from entering the labor force for the overall household, by multiplying the marginal valuation of wealth (λ_t) with the monetary value of being in the workforce, consisting of the real private-sector wage, $w_t = W_t/P_t$,

[7] Note, that in Gadatsch et al. [2016] the change in wage inflation is multiplied with w_t as in our case, while in NEMO (CITE properly) $w_t N_t$ is multiplied. This choice, however, has no first-order implications on the wage dynamics.

multiplied with the employment-rate in the private sector, the real public-sector wage, $w_t^G = W_t^G/P_t$, multiplied with the public-sector employment rate as well as the unemployment benefit multiplied with the number of household-members that are unemployed in a specific profession.[8] The right-hand side captures the disutility from being in the workforce, given by the disutility of working multiplied with probability that member j will be employed, i.e. the overall employment rate. We now define the marginal member for each profession for which this condition holds with equality as $L_t^o(i)$, where $L_t^o(i)$ can be interpreted as the profession-specific participation rate. This is because for members of index $j < L_t^o(i)$ the utility of entering the labor force will exceed the disutility, while for members of index $j > L_t^o(i)$ the reverse will be true. The participation decision in the optimizing household is then given by

$$\lambda_t \left((1-\tau_t^W)(w_t(i)N_t^{o,P}(i) + w_t^G N_t^{o,G}) + UB_t(L_t^o(i) - N_t^o(i)) \right) = N_t^o(i)\epsilon_{g,t}\theta(L_t^o(i))^{\psi}. \tag{10}$$

We will later show, that the participation rate is equal across all types of specialized work such that the i can be dropped from above equation eventually. Note, that the total employment rate N_t^o is the sum of private employment $N_t^{o,P}$ and government employment $N_t^{o,G}$. The share of household members that is unemployed is then given by $U_t^o = L_t^o - N_t^o$ with the more commonly used measure of the unemployment rate, expressing the share of household members unemployed relative to those in the workforce is given by

$$UR_t^o = \frac{L_t^o - N_t^o}{L_t^o}. \tag{11}$$

While we will present the modeling of firms in the next section, it is useful to derive the profession-specific labor demand by firms at this point since the labor demand function enters the wage-setting problem of the household members. Following Gadatsch et al. [2016], the homogeneous composite of labor input for domestic intermediate firms, N_t^P, is given by a CES aggregate over the differentiated individual labor services, i.e.

$$N_t^P = \left(\int_0^1 \left(N_t^P(i)\right)^{(\epsilon_w - 1)/\epsilon_W} di \right)^{\epsilon_W/(\epsilon_w - 1)}$$

where ϵ_W is the degree of substitutability between different (household-specific) labor inputs. Firms seek to maximize total labor input N_t^P for a given wage bill $\int_0^1 N_t^P(i)W_t(i)di =: Z(t)$. The solution of this problem is the private-sector demand for each specialized labor type i and is given by

$$N_t^P(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W} N_t^P$$

As we will discuss in more detail further below, the population consists of a optimizing and a Rule-of-thumb household who each provide labor. We assume that their (private-sector) employment rates are, however, identical, hence $N_t^{o,P}(i) = N_t^{r,P}(i) = N_t^P(i)$. Thus we can write

$$N_t^{o,P}(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W} N_t^{o,P}. \tag{12}$$

The demand for their labor that each profession within the optimizing household faces is thus a function the wage set by that particular profession, i.e. $W_t(i)$ relative to the

[8] If household member j decides to enter the labor force, she can assume that every household-member with a lower degree of disutility of labor, i.e. members of index $< j$, will also enter the labor force. As those with index $\leq N_t^o(i)$ will be employed, those of index $N_t^o(i) < \hat{j} < j$ will be unemployed.

average wage in the economy, W_t. When setting their wage, household members within a profession internalize this relationship between wages and labor demand.

Capital accumulation and utilization The evolution of the household's capital stock is given by

$$K_{t+1}^o = \left[1 - \frac{\chi_K}{2}\left(\frac{I_t^o}{I_{t-1}^o} - 1\right)^2\right] I_t^o + (1-\delta_0)K_t^o \tag{13}$$

where $\left[1 - \frac{\chi_K}{2}\left(\frac{I_t^o}{I_{t-1}^o} - 1\right)^2\right]$ denotes the share of investment I_t^o that effectively increases the capital stock. The remainder can be interpreted as investment adjustment costs $\gamma_t^K := \left(\frac{\chi_K}{2}\left(\frac{I_t^o}{I_{t-1}^o} - 1\right)^2\right) I_t^o$. The rational for this equation lies in the presence of costs of, for example, planning and putting a particular kind of investment into place.[9] These adjustment costs are an increasing function of the change in investment relative to the last period, with the parameter $\chi_K > 0$ governing the magnitude of the cost. Besides investments, the optimizing household also chooses the utilization rate of capital, u, following the model approach in Adolfson et al. [2007]. The cost (benefit) with over (under) utilization of capital is given by

$$\Gamma(u_t) = \chi_{u,1}(u_t - 1) + \frac{\chi_{u,2}}{2}(u_t - 1)^2 \tag{15}$$

where $\chi_{u,1}$ and $\chi_{u,2}$ are parameters larger than zero. A value of $u = 1$ equals the "normal" or steady-state level of capital utilization and yields zero cost of utilization. If u is chosen below 1, capital rented out to firms will be used less intensely resulting in lower output while at the same time the income from renting out the capital is also reduced. However, the term $\Gamma(u_t)$ will turn negative (which will be ensured by appropriate parameter choice) such that the household generates a benefit from capital under-utilization. This benefit can be viewed, for example, as a lower rate of capital depreciation. Conversely a higher capital utilization rate than unity causes this benefit to turn negative, i.e. $\Gamma(u_t) > 0$, while at the same time output and rental income rises. Thus, variable capital utilization provides the household with an additional instrument to react to business cycles.

Optimal decision by the optimizing household and its members Maximization of the lifetime utility in equation (1) subject to the budget constraint given by (8), to the labor force participation constraint given by (10) as well as to the capital accumulation equation given in (13) yields the Lagrangian

$$\begin{aligned}
\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \Bigg(\epsilon_{g,t}\left[\frac{(C_t^o - H_t)^{1-\sigma}}{(1-\sigma)(1-h)^{-\sigma}} - \theta \int_0^1 \frac{N_t^o(i)^{1+\psi}}{1+\psi} di + \theta_G \frac{(G_t^u)^{1-\sigma_G}}{1-\sigma_G}\right] \\
+ \; & \lambda_t \frac{1}{P_t} \left[\text{r.h.s of eq. 8 - l.h.s of eq. 8}\right] \\
+ \; & \lambda_t^L \left[\text{r.h.s of eq. 10 - l.h.s of eq. 10}\right] \\
+ \; & \mu_t \left[\text{r.h.s of eq. 13 - l.h.s of eq. 13}\right]\Bigg)
\end{aligned}$$

[9] The evolution of capital can alternatively, and potentially somewhat more intuitively, be written as

$$K_{t+1}^o = I_t^o + (1-\delta_0)K_t^o - \gamma_t^K. \tag{14}$$

where λ_t is the real shadow value of one unit of a domestic bond (or one unit of foregone consumption), λ_t^L the real shadow value of the participation rate and μ_t the real shadow value of capital. The equations in this section are based on applying the maximum principle to above problem and in particular their first-order conditions, the derivations of which can be found in the appendix 6.1.

Bonds: We first consider the optimal choice in bond holdings, both for domestic government bonds as well as for foreign bonds. Defining the after-tax nominal gross return on domestic and foreign bonds as

$$\begin{aligned} R_{t,net} &= 1 + (R_t - 1)(1 - \tau_{t+1}^{OI}) \quad (16) \\ R_{t,net}^* &= 1 + (R_t^* \phi_t - \frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t})(1 - \tau_{t+1}^{OI}) \quad (17) \end{aligned}$$

we can derive the uncovered interest parity (UIP) condition as

$$E_t \left(\lambda_{t+1}/\pi_{t+1}[R_{t,net} - R_{t,net}^* \frac{e_{t+1}}{e_t}] \right) = 0. \quad (18)$$

The UIP condition ensures that risk-free profit exploiting currency arbitrage is not possible as household will allocate their bond-wealth across foreign and domestic bonds in such a way that after-tax gross returns (adjusted for changes in the nominal exchange rate) are equal in expectation. Of particular importance in this respect is the risk premium on foreign bonds (and thus the effective after-tax gross return $R_{t,net}^*$) depends on the level of foreign bond holdings, which ensures that there is only one valid allocation of bonds, see Schmitt-Grohé and Uribe [2003]. Taxation of the return on bonds (including the taxation of capital gains due to exchange rate movements) significantly complicates the formulaiton of the UIP-condition. Abstracting from taxation, the UIP condition simplifies to the standard textbook formulation of $E_t \left(\lambda_{t+1}/\pi_{t+1}[R_t - R_t^* \phi_t \frac{e_{t+1}}{e_t}] \right) = 0$, which can be transformed to the linearized version

$$\widehat{R}_t - \widehat{R}_t^* = (1 - \chi_e) E_t \Delta \widehat{e}_{t+1} - \chi_e \Delta \widehat{e}_t - \chi_a \widehat{A}_t + \chi_O \widehat{OILR}_t + \widehat{\widehat{\phi}}_t,$$

where Δ is the first difference operator.[10]

Consumption: Next, we determine the optimal consumption rule, given by

$$\lambda_t = \frac{\epsilon_{g,t}(C_t^o - H_t)^{-\sigma}}{(1 + \tau_t^C)(1 - h)^{-\sigma}}. \quad (19)$$

Hence consumption is allocated in such a way that its marginal utility (the right-hand side of the equation) equals λ_t which captures the shadow value of one unit of domestic bond. Consumption higher than this shadow value would mean that the marginal utility of consumption would lie below the marginal utility of holding savings (in the form of a government bond), such that consumption needs to fall for optimality. Evaluating above equation at t as well as $t+1$ and diving by each other would show that the growth rate of consumption is a function of $\frac{\lambda}{\lambda_{t+1}}$ which, following (85), is a function of the after-tax gross interest rate, inflation and time-preference β. Hence the well-known Euler equation holds for the consumption growth of optimizing households. Note, however, that the dynamics of aggregate consumption in the model economy will also be shaped by Rule-of-thumb households that will be introduced later on.

[10] In the appendix, we provide a linearized form the the UIP condition that includes bond taxation.

Wages: A further decision variable in the model are wages. Here, the unit at which decisions are made are single professions. Each profession maximizes the household's overall utility under consideration of the labor demand function of firms, see equation (12), as well as the profession-specific labor-force participation constraint, see equation (10). Solving this problem yields

$$\begin{aligned}(1-\tau_t^W)(1-\epsilon_W)w_t &= -\frac{\epsilon_{g,t}\theta(N_t^o)^\psi\epsilon_W}{\lambda_t} - UB_t\epsilon_W + \frac{DACW_t}{N_t^{o,P}}w_t - \beta\frac{\lambda_{t+1}}{\lambda_t}DACW_{t+1}w_{t+1}\frac{1}{N_t^{o,P}} \\ &\quad -\frac{\lambda_t^L}{\lambda_t}\left(-\epsilon_W\epsilon_{g,t}\theta(L_t^o)^\psi\right) + \lambda_t^L\left((1-\tau_t^W)(1-\epsilon_W)w_t + UB_t\epsilon_W\right) \qquad (20)\end{aligned}$$

where $DACW_t$ captures the differential of wage adjustment costs and is defined in the Appendix. The equation above is best understood by first considering a simplified model environment, namely one where unemployment benefits are set to zero, wage renegotiation is not subject to adjustment costs as well as labor force participation is fixed ($\lambda_L = 0$). The wage-setting equation then collapses to the standard wage-setting equation found in Galí et al. [2012] augmented by taxation.

$$(1-\tau_t^W)w_t = \frac{\epsilon_{g,t}\theta(N_t^o)^\psi}{\lambda_t}\frac{\epsilon_W}{\epsilon_W - 1} \qquad (21)$$

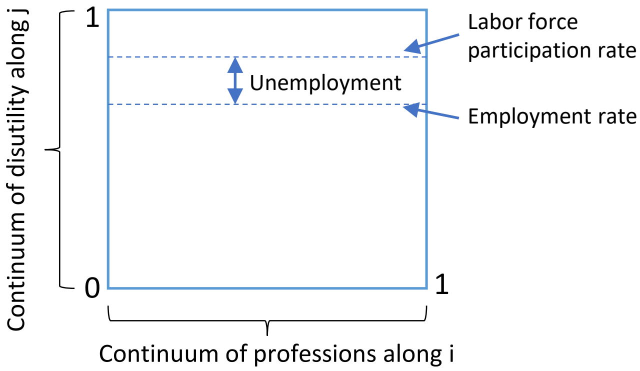
Hence, the wage is set such the after-tax wage rate equals the marginal rate of substitution between consumption and labor ($\frac{\epsilon_{g,t}\theta(N_t^o)^\psi}{\lambda_t}$) multiplied with the wage mark-up resulting from imperfect substitution across professions, namely $\frac{\epsilon_W}{\epsilon_W-1}$, which will be larger than unity in our calibration. Due to the fact that wages are set with a mark-up over the marginal rate of substitution between consumption and labor, more household members chose to be in the labor force than labor is demanded such that involuntary employment arises, see Galí et al. [2012]. The intuition behind this result is that in a perfectly competitive labor market economy, wages would adjust such that labor demand and supply exactly clears, which is at the marginal rate of substitution. If wages are set above this perfect competition benchmark, the supply of labor will exceed its demand leaving some in the labor force unemployed.

Returning to equation (20), the presence of unemployment benefits ceteris paribus dampens the negative implications of unemployment to the household such that wages can be set higher. The introduction of a labor force participation decision is less trivial to understand. Intuitively speaking, wage-setters have now take into account that the wage-level influences not only the number of members unemployed but also the number of members in the labor-force which affects the utility of the overall household through missed-out unemployment benefits.

An important conclusion is to be drawn from equation (20): Note, that the wage set does not depend on the profession. As can be seen in the appendix, the optimization problem across professions is symmetrical such that all profession choose the same wage level, and thus, the exhibit the same employment and participation rate. Adapting the graphical representation from figure 2, figure 3 illustrates the consequences of this finding.

Given the negotiated wage level, firms demand a certain level of labor determining the employment rate. Given these and the level of unemployment benefits members decide whether to participate in the labor force and thereby determine the labor force participation rate. Only those with the highest level of disutility across household members will chose to stay out of the labor force, i.e. those members above the upper dashed line will in

Figure 3: Representation of the optimizing household



the figure. The higher the level of wages and unemployment benefits in the economy the higher this upper dashed lined will be. As long as wages are set with an mark-up over the marginal rate of substitution between consumption and employment, the employment rate will lie below the labor force participation rate. To minimize disutility, those household member with the lowest degree of disutility will be employed while those in between the dashed lines, i.e. those whose disutility is low enough to want to work at the prevailing wage and high enough such that there are sufficiently many other members with a lower disutility to fill all jobs, remain involuntary unemployed. Note, that the dashed lines are perfectly horizontal as there is no difference in employment and labor force participation rates across professions. Business cycles will move both rates up and down. For example, an expansion in production will cause labor demand to increase and thus unemployment to fall. After some time wages will increase incentivizing those out of the labor force to seek jobs.

Capital and Investment: Ignoring variable capital utilization for now, the first-order condition on the level of capital is given by

$$\mu_t = \beta E_t(\lambda_{t+1}[(1-\tau^{OI}_{t+1})R^k_{t+1} + \tau^{OI}_{t+1}P^i_{t+1}\delta_0] + \mu_{t+1}(1-\delta_0))$$

where μ_t denotes the marginal utility of capital. Thus, the marginal utility of capital at time t rises with the expected after-tax (including the tax rebate on capital depreciation) rental rate on capital in the next period as well as with the expected marginal utility of capital in the next period adjusted for depreciation. Variable capital utilization enables the household to influence the rental income on their capital. The full first-order condition in the appendix captures this effect as well. The investment decision is determined by the corresponding first-order condition given in (94). A more intuitive approach to understanding the investment decision of the household is provided by Tobin's q, defined as

$$q_t = \frac{\mu_t}{\lambda_t P^i_t}. \tag{22}$$

where $\frac{\mu_t}{\lambda_t}$ is to be interpreted shadow price of capital, as the ratio of the marginal utility of capital and the marginal utility of consumption yields the value of a marginal unit of capital in terms of a consumption unit. When the price of installed capital in terms of a consumption unit exceeds the replacement cost in terms of a consumption unit, i.e. P^i_t, or in other words when $q > 1$, firms' incentive to invest increases.[11] The decision to invest

[11] Note a relationship between investment and Tobin's q only exists if there are adjustment costs associated with the accumulation of capital. This is because q_t equals 1 at all times if adjustment costs are set to

thus depends on the the current price of investment and the expected value of installed capital, which itself depends on expected after-tax return on capital.
Finally, the optimal level of capital utilization is given by

$$u_t = \left(\frac{R_t^k}{P_t^i} - \chi_{u,1}\right) / \chi_{u,2} + 1,$$

implying that capital utilization increases in the rental rate of capital, to take advantage of high income on capital, and falls in the price of investment as capital over-utilization is payed for in terms of investments good.

2.2.2 Rule-of-Thumb household

A second household in the economy is characterized by rule-of-thumb (RoT) behaviour, implying that the household consumes all of its income net of taxes and transfers. Thus, this household does not save in form of bonds or capital nor does it own any firms. The interpretation of this behaviour include among other the lack of access to capital markets, myopia and ignorance or inattention to intertemporal trading opportunities. More importantly, the inclusion of Rule-of-Thumb households breaks the so-called Ricardian equivalence, i.e. the internalization of the government budget constraint in the saving decision of the households in an economy. Contrary to such Ricardian behaviour, RoT households will not anticipate that a change in taxation today will cause an equally large opposite change in taxation in terms of present discounted value some time in the future, but will immediately adjust their consumption. Several studies, including Galí et al. [2007] and Campbell and Mankiw [1989], highlight the quantitative importance of such RoT consumers among industrialized economies.

We model the RoT household along the lines of Galí et al. [2007]. The budget constraint (in nominal terms) is thus given by

$$P_t C_t^r (1 + \tau_t^C) = (1 - \tau_t^W)(W_t N_t^{r,P} + W_t^G N_t^{r,G}) + P_t U B_t (L_t^r - N_t^r) + P_t T R_t^r \tag{23}$$

where the variables with superscript "r" are simply the RoT analog to the same variables already introduced for the optimizing household with superscript "o". Hence total expenditures of the RoT household consist only of consumption expenditures (left-hand side of the equation), while income is generated from employment in both the public and private sector as well as other income from public sources, namely unemployment benefits as well as transfers. Note, that tax rates and level of unemployment benefits does not differ across the two types of household. However, the model allows for different level of transfers.

The rule-of-thumb household adopts the wage level, the employment rates as well as the labor-force participation rate set by the optimizing household. We thus set

$$N_t^{r,P} = N_t^{o,P} \tag{24}$$
$$N_t^{r,G} = N_t^{o,G} \tag{25}$$
$$N_t^r = N_t^o \tag{26}$$
$$L_t^r = L_t^o. \tag{27}$$

zero as follows from equation (94).

2.2.3 Household aggregation

To conclude the section on households, we need to define the aggregate measures of household variables. To do so we first define the population size to be 1, with the size of the RoT household amounting to $0 \leq \omega < 1$ and the complementary size of the optimizing household being $0 < 1 - \omega \leq 1$.

For those household variable that occur in both household types, aggregation is thus given by

$$
\begin{aligned}
C_t &= \omega C_t^r + (1-\omega)C_t^o, && (28)\\
N_t^P &= \omega N_t^{r,P} + (1-\omega)N_t^{o,P} = N_t^{r,P} = N_t^{o,P}, && (29)\\
N_t^G &= \omega N_t^{r,G} + (1-\omega)N_t^{o,G} = N_t^{r,G} = N_t^{o,G}, && (30)\\
N_t &= \omega N_t^r + (1-\omega)N_t^o = N_t^r = N_t^o && (31)\\
L_t &= \omega L_t^r + (1-\omega)L_t^o = L_t^r = L_t^o, && (32)\\
TR_t &= \omega TR_t^r + (1-\omega)TR_t^o, && (33)
\end{aligned}
$$

denoting aggregate household consumption, private-sector, public-sector and total employment rate, labor force participation rate as well as total transfers.

For those variables that only occur within the household type of optimizers, we nevertheless have to weigh the variable with the size of the optimizing household to arrive at the aggreagte measure that can be used in the market clearing conditions, i.e.

$$X_t = (1-\omega)X_t^o \tag{34}$$

for $X \in \{I, D, B, K, T^L, DIV, \gamma\}$.

2.3 Firms

The structure of the production side of the economy is based on the set-up in Justiniano and Preston [2010], in which the economy can be subdivided intro three parts.

First, the intermediate good sector uses domestic labor and capital to produce the intermediate good, which can either be sold abroad or sold domestically to the final good sector or the government. Labor is supplied by wage-setting households while, following Coenen et al. [2013], capital is a composite function of private capital, supplied by households, and public capital which is provided by the government. The firms pay rent to households for capital services and wages for the supplied labor. The government capital is available at no direct cost to firms. However, firms are subject to taxation. The firms are assumed to be monopolistically competitive and sell their product at different prices domestically and internationally.

Second, importing firms purchase the foreign good at the world market price and sell the foreign good as an intermediate good domestically. The importing firms are assumed to be monopolistically competitive and thus set prices at a markup over their marginal cost.

Third, the final good sector combines domestic and imported intermediate goods to produce the final consumption and investment goods purchased by households, the price of which are P_t and P_t^I, respectively.[12]

[12]Note, that we differ from the set-up in Justiniano and Preston [2010] in several regards. First, our model

We introduce the following definitions. The price of the domestically produced intermediate good is given by $P_{H,t}$, the price of the imported intermediate good by $P_{F,t}$. The overall CPI of the economy is given by P_t, such that $P_{h,t} = P_{H,t}/P_t$ and $P_{f,t} = P_{F,t}/P_t$ are the relative prices of the home and foreign intermediate good using the CPI as the numeraire. CPI inflation, home good and foreign good inflation are given by, $\pi_t = P_t/P_{t-1}$, $\pi_{H,t} = P_{H,t}/P_{H,t-1}$ and $\pi_{F,t} = P_{F,t}/P_{F,t-1}$, respectively. We define the terms of trade as the price ratio beween the exported and imported good, i.e. $S := \frac{QP^*_{x,t}}{P_{f,t}}$.

2.3.1 Final good sector

We begin the description of the production structure with the final good sector, which consists of two parts, the production of the final consumption and investment good.

Final consumption good: The production of the final consumption good uses domestically produced consumption goods, $C_{H,t}$, and those imported, $C_{F,t}$, and follows the following production function:

$$C_t = \left[(1-\alpha_C)^{1/\eta_c}(C_{H,t})^{\frac{\eta_c-1}{\eta_c}} + \alpha_C^{1/\eta_c}(C_{F,t})^{\frac{\eta_c-1}{\eta_c}}\right]^{\eta_c/(\eta_c-1)} \tag{35}$$

where α_C is the home bias parameter in consumption and η_c is the elasticity of substitution between the imported and domestically produced consumption good.[13] Given the prices for domestic and imported consumption goods, the final good sector seeks to minimize its costs of inputs to production (i.e. imported and domestic consumption goods) for a certain desired level of production C_t. The derivation of the problem is given in Appendix 6.2, yielding the following result:

$$C_{H,t} = (1-\alpha_C)(P_{h,t})^{-\eta_c} C_t, \tag{36}$$
$$C_{F,t} = \alpha_C (P_{f,t})^{-\eta_c} C_t. \tag{37}$$

The consumer price index (i.e. the pre-tax price of one unit of the final consumption good) is then given by

$$P_t = \left((1-\alpha_C)(P_{H,t})^{1-\eta_c} + \alpha_C (P_{F,t})^{1-\eta_c}\right)^{1/1-\eta_c}. \tag{38}$$

Final investment good Total investment demand in the economy is given by

$$\widehat{I}_t = \underbrace{I_t}_{\text{Inv. by households}} + \underbrace{\Gamma(u_t)K_t}_{\text{Capital utilization costs}} + \underbrace{I_t^G}_{\text{Gov. investment}} + \underbrace{I_t^{OIL}}_{\text{Oil sector investment}}. \tag{39}$$

Analogous to the consumption case, the total investment demand is satisfied by a composite of foreign and domestic investment, i.e.

features distinct prices for goods sold domestically and abroad. Second, we introduce a range of taxes affecting the intermediate good producer and importers. Third, capital employed in production is a composite of public and private capital as ain Coenen et al. [2013]. Fourth, the final good sector in Justiniano and Preston [2010] is conceptualized as households bundling domestic and imported goods. While the latter point has no implications for the model dynamics, the first three points will affect the transmission of shocks beyond the mechanisms in Justiniano and Preston [2010].

[13] Note, that since the consumption tax is levied on the composite consumption good C_t (instead of on $C_{H,t}, C_{F,t}$) in the budget constraint of the households, we implicitly assume that the domestically produced and the imported consumption good is taxed at the same rate.

$$\widehat{I}_t = \left[(1-\alpha_I)^{1/\eta_I}(I_{H,t})^{\frac{\eta_I-1}{\eta_I}} + \alpha_I^{1/\eta_I}(I_{F,t})^{\frac{\eta_I-1}{\eta_I}}\right]^{\eta_I/(\eta_I-1)} \tag{40}$$

where α_I is the home bias parameter in private investment and η_I is the elasticity of substitution between the imported and domestically produced investment good. Minimization of costs yields

$$I_{H,t} = (1-\alpha_I)(P_{h,t})^{-\eta_I}\left(P_t^i\right)^{\eta_I}\widehat{I}_t \tag{41}$$

$$I_{F,t} = \alpha_I(P_{f,t})^{-\eta_I}\left(P_t^i\right)^{\eta_I}\widehat{I}_t \tag{42}$$

where the price of investment is given by

$$P_t^I = \left((1-\alpha_I)(P_{H,t})^{1-\eta_I} + \alpha_I(P_{F,t})^{1-\eta_I}\right)^{1/1-\eta_I}. \tag{43}$$

2.3.2 Intermediate good sector

A continuum of domestic firms $i \in [0,1]$ populate the intermediate good sector and produce an intermediate good that is either exported to the foreign economy or sold domestically to the government or the final good sector. Hence, total production of firm i, $Y_{H,t}(i)$, is given by

$$Y_{H,t}(i) = Y_{H,t}^D(i) + X_t(i) \tag{44}$$

where $Y_{H,t}^D(i)$ denotes the volume of production of firm i that is sold domestically and $X_t(i)$ the volume of production that is exported. Domestically sold output and exports are produced with the same production function such that the marginal cost, which is derived in the following, is identical across these two types of outputs. However, firm (i) sets prices for domestically used output and exports separately, which will be derived futher below.

Cost minimization and marginal cost Following Coenen et al. [2013], the production function in firm i is given by

$$Y_{H,t}(i) = \epsilon_{a,t}(\widetilde{K}_t(i))^{\alpha}(N_t^P(i))^{1-\alpha}. \tag{45}$$

where α is the capital share in domestic production, $N_t^P(i)$ is the employment rate and $\widetilde{K}_t(i)$ is the composite capital stock available to firm i, consisting of of private capital $K_t(i)$ and the degree by which the firm uses public capital, $K_{G,t}(i)$, which is available at no cost. The composite capital for each firm is given by

$$\widetilde{K}_t(i) = \left[(1-\alpha_K)^{1/\eta_K}(u_tK_t(i))^{\frac{\eta_K-1}{\eta_K}} + \alpha_K^{1/\eta_K}(K_{G,t}(i))^{\frac{\eta_K-1}{\eta_K}}\right]^{\eta_K/(\eta_K-1)} \tag{46}$$

where α_K is the bias towards public capital in the capital composite and η_K denotes the elasticity of substitution between private and public capital.[14] Intermediate good producers minimize their total cost given the production function and the capital composite function. The problem is given by

$$\min_{N_t^P(i),K_t(i)} TC_t(i) = \min_{N_t^P(i),K_t(i)} (1+\tau_t^{SS,F})w_tN_t^P(i) + u_tK_t(i)R_t^k$$

[14] The case $\eta_K \to 0$ implies perfect complements, $\eta_K \to \infty$ gives perfect substitutes and $\eta_K \to 1$ yields the Cobb-Douglas specification.

where $\tau_t^{SS,F}$ denotes the social security tax paid by firms. The problem is derived and solved in 6.3. The optimal input conditions are given by

$$\frac{(1+\tau_t^{SSF})w_t}{R_t^k}\frac{\alpha}{1-\alpha}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_tK_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{-1}{\eta_K}} = \frac{\widetilde{K}_t(i)}{N_t^P(i)}. \tag{47}$$

Marginal cost of firm i can be derived as

$$MC_t(i) = \frac{(R_t^k)^{\alpha}((1+\tau_t^{SS,F})w_t)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\epsilon_{a,t}}\frac{\left[(1-\alpha)+\alpha(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_tK_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}}\right]}{(1-\alpha_K)^{\frac{\alpha}{\eta_K}}\left(\frac{u_tK_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{-\alpha}{\eta_K}}}. \tag{48}$$

We assume, that each firm exhibits the same ratio of public and private capital used in each period, i.e. that $\frac{K_{G,t}(i)}{K_t(i)}$ is independent of i. As a consequence, the ratio $\frac{u_tK_t(i)}{\widetilde{K}_t(i)}$ is constant across all firms at each point in time, such that marginal cost is independent of the firm and its size.[15] Note also, that in the absence of public capital, i.e. $\alpha_K = 0$ and $u_tK_t(i) = \widetilde{K}_t(i)$ the expression for optimal factor inputs and the marginal cost collapses to the standard formulation for Cobb-Douglas production functions involving only the rental rate for capital, the wage rate and the production elasticity.
Finally we define the mark-up as

$$\mu_t^h = \frac{P_t^H/P_t}{MC_t} = \frac{P_{h,t}}{MC_t}. \tag{49}$$

Price setting for domestic output Single firms produce the output $Y_{H,t}^D(i)$ sold at the price $P_{h,t}(i)$. The total demand of domestically produced goods $Y_{H,t}^D$ is, however, produced by perfectly-competitive retailers[16] who buy the output of single firms and combine it according to the function

$$Y_{H,t}^D = \left(\int_0^1 Y_{H,t}^D(i)^{\frac{\epsilon_h-1}{\epsilon_h}}di\right)^{\frac{\epsilon_h}{\epsilon_h-1}}, \tag{50}$$

where ϵ_h is the elasticity of substitution of the intermediate good across firms. Given any level of expenditures $\int_0^1 P_{h,t}(i)Y_{H,t}^D(i)di$, the retailers aim to maximize (50) which yields the set of demand equations given by[17]

$$Y_{H,t}^D(i) = \left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\epsilon_h}Y_{H,t}^D.$$

Hence, how much the retailer will demand from a particular firm, $Y_{H,t}^D(i)$, will depend on the price that firm sets relative to the aggregate price index $P_{h,t} = \int_0^1 P_{h,t}(i)^{1-\epsilon_h}di^{\frac{1}{1-\epsilon_h}}$. The pricing problem of individual firms is then given as follows. After-tax profits of firm i related to domestically used output net of the corporate tax $\tau_t^{\Pi,F}$ are given by

[15]Note, that this ratio is not constant across time but only across firms at each point in time. Depending on the economic dynamics the ratio will change over time, but in the same way for all firms. For example, newly created public infrastructure prompts all firms to increase the amount of (private) capital rented. The increase might not be proportional to the increase in public capital stock such that the ratio changes. However, all firms will increase their capital stock in a way, that the ratio of private to public capital changes to the same value for all firms after the expansion.

[16]We have not mentioned this entity in our model introduction due to its rather limited role in production.

[17]The derivation to this problem can be found in any basic textbook on New-Keynesian DSGE models and is left out for this reason.

$$(1-\tau_t^{\Pi,F})\Pi_{H,t}(i) = (1-\tau_t^{\Pi,F})\left[(P_{h,t} - MC_t)Y_{H,t}^D(i) - AC_{H,t}(i)\right] \tag{51}$$

where adjustment costs are given by

$$AC_{H,t}(i) = \frac{\chi_h}{2}\left(\frac{\frac{P_{h,t}(i)}{P_{h,t-1}(i)}\pi_t}{\left(\frac{P_{h,t-1}}{P_{h,t-2}}\pi_{t-1}\right)^{\chi_a}\overline{\pi}^{1-\chi_a}} - 1\right)^2 Y_{H,t}^D P_{h,t}. \tag{52}$$

Hence, $AC_{H,t}(i)$ denotes adjustment cost in real terms (in terms of the CPI) for the overall domestically used production of firm i.[18] Note, that $\frac{P_{h,t}(i)}{P_{h,t-1}(i)}\pi_t$ is simply another way of writing $\frac{P_{H,t}(i)}{P_{H,t-1}(i)}$. Hence, adjustment cost operate on price inflation of the home good. The price-setting equation and its derivation is provided in the Appendix 6.4. The result implies that domestic firms set the same price, i.e. $P_{h,t}(i) = P_{h,t}$, which in the steady state equals the $MC_t\frac{\epsilon_h}{\epsilon_h-1}$. Hence, the price of domestically produced goods sold domestically is set with an mark-up over production costs.

Price setting for exports The prices at which exports are sold to the foreign economy are set in the foreign currency itself. The foreign economy's demand for the domestic intermediate good is given by the function

$$X_t = \left(P_{x,t}^*\right)^{-\eta_x} Y_t^* \tag{53}$$

with $P_{x,t}^* = \frac{P_{X,t}^*}{P_t^*}$ is the price of the homogenous export good in foreign currency relative to the foreign CPI price level, the latter being exogenously given in the model. Y_t^* denotes world output and will be discussed in section 2.5. As previously, the single domestic firms are not providing aggregate exports directly but sell their intermediate goods to export retailers who produce an homogenous export good which is then provided to the foreign sector subject to the demand function (53). A single firm i generates exports of volume $X_t(i)$ selling at price $P_{x,t}^*(i)$. The export retailers operate under perfect competition and bundle the output of single firms according to

$$X_t = \left(\int_0^1 X_t(i)^{\frac{\epsilon_x-1}{\epsilon_x}} di\right)^{\frac{\epsilon_x}{\epsilon_x-1}}.$$

where ϵ_x is the elasticity of substitution of the intermediate export good across the firms. Output maximization analogous to the case of domestic firms then implies

$$X_t(i) = \left(\frac{P_{x,t}^*(i)}{P_{x,t}^*}\right)^{-\epsilon_x} X_t. \tag{54}$$

implying that the export demand an individual firm is facing is a negative function of the price it sets relative to the aggregate price index $P_{x,t}^* = \int_0^1 P_{x,t}^*(i)^{1-\epsilon_x} di^{\frac{1}{1-\epsilon_x}}$. The pricing problem of individual firms is then given as follows. Profits of firm i related to export output net of the corporate tax are given by

$$(1-\tau_t^{\Pi,F})\Pi_{X,t}(i) = (1-\tau_t^{\Pi,F})[(P_{x,t}^*(i)Q_t - MC_t)X_t(i) - AC_{X,t}(i)]. \tag{55}$$

Adjustment costs are given by

[18] It is not the adjustment cost per unit of production and hence not multiplied with the amount of production in the profit function. Instead $AC_{H,t}(i)$ enter the profit function simply as an additional cost, that is already expressed in CPI terms.

$$AC_{X,t}(i) = \frac{\chi_x}{2}\left(\frac{\frac{P^*_{x,t}(i)}{P^*_{x,t-1}(i)}\pi^*_t}{\left(\frac{P^*_{x,t-1}}{P^*_{x,t-2}}\pi^*_{t-1}\right)^{\chi_a}(\bar{\pi}^*)^{1-\chi_a}} - 1\right)^2 X_t Q_t P^*_{x,t}, \tag{56}$$

respectively. Hence, $AC_{X,t}(i)$ denotes adjustment cost in real domestic currency terms for the exporter i. The solution to the price-setting problem, namely the maximization of the net present value of (55) subject to the demand (54), is provided in Appendix 6.5. Most importantly, the result reveals that export prices in the steady state are set at an mark-up of $\frac{\epsilon_x}{\epsilon_x - 1}$ over marginal cost and all firms set identical prices such that $P^*_{x,t}(i) = P^*_{x,t}$. The identify of export and domestic prices across firms also implies identity of profits, employment rates and capital employed across firms such that we can drop the (i)-dependence for these variables.

2.3.3 Import sector

The total demand of imports consists of the demand for imported consumption goods $C_{F,t}$, which is given in equation (37) and the demand of imported investment goods, $I_{F,t}$, which is given in equation (42). Total imports are then given by

$$IM_t = C_{F,t} + I_{F,t}. \tag{57}$$

As previously, the single firms are not providing aggregate imports directly but sell their intermediate import goods to import retailers who produce an homogenous import good which is then provided to the final good sector. A single import firm i generates output of volume $IM_t(i)$ selling at price $P_{f,t}(i)$. The retailer operate under perfect competition and bundle the output of single import firms according to

$$IM_t = \left(\int_0^1 IM_t(i)^{\frac{\epsilon_f - 1}{\epsilon_f}} di\right)^{\frac{\epsilon_f}{\epsilon_f - 1}}.$$

where ϵ_f is the elasticity of substitution of the intermediate import good across the firms. Output maximization analogous to the case of domestic firms then implies

$$IM_t(i) = \left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\epsilon_f} IM_t. \tag{58}$$

implying that the demand an individual import firm is facing is a negative function of the price it sets relative to the aggregate price index $P^f_t = \int_0^1 P^f_t(i)^{1-\epsilon_f} di^{\frac{1}{1-\epsilon_f}}$. The pricing problem of individual firms is then given as follows. After-tax profits of the importing firm i at time t are given by

$$(1 - \tau^{\Pi,F}_t)\Pi_{F,t}(i) = (1 - \tau^{\Pi,F}_t)(P_{f,t}(i) - Q_t)IM_t(i) - AC_{F,t}(i) \tag{59}$$

where $\tau^{\Pi,F}_t$ is the corporate tax. Note, that the "cost of production" of one unit of output equals the real exchange rate since this is the price at which the importer can purchase one unit of foreign output. Adjustment costs are given by

$$AC_{F,t}(i) = \frac{\chi_f}{2}\left(\frac{\frac{P_{f,t}(i)}{P_{f,t-1}(i)}\pi_t}{\left(\frac{P_{f,t-1}}{P_{f,t-2}}\pi_{t-1}\right)^{\chi_a}\bar{\pi}^{1-\chi_a}} - 1\right)^2 IM_t P_{f,t}. \tag{60}$$

Hence, $AC_{H,t}(i)$ and $AC_{F,t}(i)$ are defined perfectly analogous. The price-setting problem then consists of maximizing net present value of profits given in (59) subject to the demand constraint given by the retailers (58) and is given in Appendix 6.6. The result implies that all import firms set the same price, i.e. $P_t^f(i) = P_t^f$, which in the steady state equals the $Q_t \frac{\epsilon_f}{\epsilon_f - 1}$. Hence the selling price is set with an mark-up over the purchasing price.

2.3.4 Profits and Dividends

Total firm profits are given as the sum of profits over domestic and export sales minus costs of production and price adjustment costs, i.e.

$$\begin{aligned} \Pi_t &= \Pi_{H,t} + \Pi_{X,t} = P_{h,t} Y_{H,t}^D + Q_t P_{x,t}^* X_t - TC_t - AC_{H,t} - AC_{X,t} \\ &= P_{h,t} Y_{H,t}^D + Q_t P_{x,t}^* X_t - (1 + \tau_t^{SS,F}) w_t N_t^P - R_t^k u_t K_t - AC_{H,t} - AC_{X,t} \end{aligned} \tag{61}$$

After-tax profits of firms in the intermediate good sector are redistributed as dividends to households:

$$DIV_t = \Pi_t (1 - \tau_t^{\Pi,F}) \tag{62}$$

Profits that accrue in the importing sector are assumed to remain in the foreign economy. The final good sector as well as retailers are perfectly competitive and thus generate no profits.

2.4 Monetary and Fiscal Policy

The modeling of the monetary and fiscal policy follows in many ways standard NK-DSGE models. On the fiscal side we introduce, however, a large number of spending and revenue instruments that by far exceed most fiscal set-ups in DSGE models. Examples similar in the level of fiscal detail are Gadatsch et al. [2016] and Stähler and Thomas [2012].

2.4.1 Central bank

The central bank sets the nominal interest rate according to the following rule

$$R = \overline{R} \left(\frac{R_{t-1}}{\overline{R}} \right)^{\psi_r} \left(\left(\frac{\pi_t}{\overline{\pi}} \right)^{\psi_p} \left(\frac{Y_t}{\overline{Y}} \right)^{\psi_y} \right)^{1-\psi_r} \exp(\epsilon_r) \tag{63}$$

where $\overline{X}$ denotes the steady-state value of the corresponding variable X_t. The parameters ψ_r, ψ_p and ψ_y denote the weight on the smoothing of the interest rate, inflation rate and output, respectively. The term ϵ_r reflects a shock to the monetary policy rule.

2.4.2 Government budget

The government finances its expenditures consisting of consumption of domestically produced goods and services, public investments, unemployment benefits, transfers to households, the public employment wage bill as well as debt servicing, by withdrawals from public assets[19] and a range of tax instruments, of which table 1 provides an overview on. Total government revenue is thus given by

[19] Note, that these withdrawals are at the moment not connected to a depletable fund. Hence, any changes in the take-out rate from this fund will have no consequences on the government budget in the long-run, which is unrealistic. A more realistic framework of the use of public assets will follow in a future version of the model.

Table 1: Overview on tax instruments

Variable	Description	applied to
τ_t^C	Consumption tax	Households
τ_t^{OI}	Ordinary income tax	Households
τ_t^{BT}	Bracket tax	Households
$\tau_t^{SS,H}$	Social security tax	Households
T_t^L	Lump-sum tax	Households
$\tau_t^{SS,F}$	Social security tax	Firms
$\tau_t^{\Pi,F}$	Corporate tax	Firms

$$
\begin{aligned}
T_t = \quad & \underbrace{T_t^L}_{\text{Lump-sum tax}} + \underbrace{C_t\tau_t^C}_{\text{Consumption tax}} + \underbrace{(w_t N_t^P + w_t^G N_t^G)(\tau_t^{OI} + \tau_t^{BT} + \tau_t^{SS,H} + \tau_t^{SS,F})}_{\text{Labor income and social-security tax}} \\
& + \underbrace{K_t[u_t R_t^k - (\delta_0 + \Gamma(u_t))P_t^i]\tau_t^{OI}}_{\text{Capital income tax - allowances}} + \underbrace{(\Pi_{H,t} + \Pi_{X,t})\,\tau_t^{\Pi,F}}_{\text{Corporate tax}} + \underbrace{DIV_t\tau_t^{OI}}_{\text{Dividend tax}} \\
& + \underbrace{\left[\frac{P_{t-1}}{P_t}D_{t-1}(R_{t-1}-1) + e_t\frac{P_{t-1}^*}{P_t}B_{t-1}\left(R_{t-1}^*\phi(A_{t-1}) - \frac{e_{t-1}}{e_t}\frac{P_t}{P_{t-1}}\right)\right]\tau_t^{OI}}_{\text{Tax on returns to bonds}}. \qquad (64)
\end{aligned}
$$

Total government expenditures are given by

$$
\begin{aligned}
G_t = \quad & \underbrace{P_{h,t}C_t^G}_{\text{Government purchases}} + \underbrace{P_t^i I_t^G}_{\text{Government investment}} + \underbrace{UB_t(L_t - N_t)}_{\text{Unemployment benefits}} \\
& + \underbrace{TR_t}_{\text{Lump-sum transfers}} + \underbrace{w_t^G N_t^G(1 + \tau_t^{SS,F})}_{\text{Government wage bill}}. \qquad (65)
\end{aligned}
$$

The government budget constraint is then given by

$$
\underbrace{T_t}_{\text{Revenue}} + \underbrace{OILR_t}_{\text{Withdrawals from public assets}} = \underbrace{G_t}_{\text{Government spending}} + \underbrace{R_{t-1}D_{t-1}\frac{1}{\pi_t} - D_t}_{\text{Government surplus}} \qquad (66)
$$

where the left (right) hand side (of the equal sign) contains real government revenue (spending) at time t.[20] Note, that the we do not model the foreign assets held by the government but only the withdrawals from it.

We define both, the government surplus, GS_t, as well as an non-oil (excluding $OILR_t$) government surplus given by , GS_t^{adj}

$$
\begin{aligned}
GS_t &= R_{t-1}D_{t-1}\frac{1}{\pi_t} - D_t \qquad (67) \\
GS_t^{adj} &= GS_t - OILR_t \qquad (68)
\end{aligned}
$$

Both, government expenditures and revenue are subject to policy-rules, implying that the government in the model is not optimally choosing these components according to welfare

[20] Government surplus is on the spending side of the equation since the government has to "spend" the surplus to reduce the debt.

measure but follows simple rules known to everybody in the economy. These rules state how revenue and expenditure react to deviations of the debt-to-GDP ratio to its target value and how smoothly they change over time in response to shocks. We use the following two policy rules:

$$\frac{X_t}{\overline{X}} = \left(\frac{X_{t-1}}{\overline{X}}\right)^{\rho_X} \left(\frac{D_{t-1}/Y_{t-1}^{CPI}}{\overline{D}/\overline{Y}^{CPI}}\right)^{(1-\rho_X)\phi_X} \epsilon_{X,t} \tag{69}$$

which applies to $X \in \{C_t^G, T_t^L, OILR_t, TR_t^r, TR_t^o, UB_t, N_t^G\}$ while an analogous but additive rules applies for the tax rates, given by

$$X_t = \overline{X} + \rho_X(X_{t-1} - \overline{X}) + (1-\rho_X)\phi_X \left(\frac{D_{t-1}}{Y_{t-1}^{CPI}} - \frac{\overline{D}}{\overline{Y}^{CPI}}\right) + \epsilon_{X,t} \tag{70}$$

where $X \in \{\tau_t^C, \tau_t^{OI}, \tau_t^{BT}, \tau_t^{SS,H}, \tau_t^{SS,F}, \tau_t^{\Pi,F}\}$. The variable Y^{CPI} denotes GDP in CPI units and will be introduced below. The policy rule associated with spending through public investment is discussed further below. The reason why the upper set of variables are expressed by a multiplicative version of the policy rule is that the shock $\epsilon_{X,t}$ can then be interpreted as percentage change in X, while tax rates are already expressed in percentage such that $\epsilon_{X,t}$ in the additive policy rule is to be interpreted as percentage point change of X. The parameters ρ_X govern smoothness of the policy rule while ϕ_X measures the responsiveness of the fiscal instrument to deviations in the debt ratio from its long-run target, namely the steady-state value of the debt-to-GDP ratio. Note, that in order to obtain non-explosive government debt ϕ_X has to be sufficiently large (i.e. positive) for at least one revenue instrument or sufficiently small (i.e. negative) for at least one of the spending instruments, as otherwise government debt might not return to its steady state in response to a shock.

This framework allows the study of the macroeconomic effects of various tax hikes and cuts or changes in government spending by shocking the corresponding tax rule. This shock may be temporary and thus only active for a limited number of periods before letting the government instrument return to its long-run target or it can be permanent such that the transition of the economy to a new long-run state can be analyzed.

2.4.3 Public investment and capital

We model the public capital stock adopting the time-to-build specification as in Leeper et al. [2010] and Coenen et al. [2013] such that authorized public investment programs exhibit a certain delay until completed and available as public capital to domestic firms. The accumulation of public capital is given by

$$K_{G,t+1} = (1-\delta_{K_G})K_{G,t} + \kappa A_{t-N+1}^{I^G} \tag{71}$$

where δ_K is the depreciation rate of public capital and κ is the share of public-investments that effectively increase the public capital stock. Hence, the model can consider both, wasteful (by setting this parameter to zero) or productive (by setting the parameter to larger than zero) public investments. The term $A_{t-N+1}^{G_I}$ denotes the authorized amount of investment $N-1$ periods ago. Hence, authorized investments become productive (unless κ is set to zero) after a time-lag of N periods. The amount of authorized investments follows an AR(1) process as given by

$$\frac{A_t^{I^G}}{\overline{A^{I^G}}} = \left(\frac{A_{t-1}^{I^G}}{\overline{A^{I^G}}}\right)^{\rho_A} \exp(\epsilon_{A,t}) \tag{72}$$

where $\overline{A^{IG}}$ is the steady-state level of authorized investment. Total spending outlays for public investment in period t are a function of the authorized investments of the last N periods. Each of these N many authorized investments is partly payed for in the current period t. We assume that all authorized investments are financed following the same schedule, characterized by the spending weights ϕ_n. The spending weight ϕ_n indicates which share of the total authorized investment expenditure is spent in the n-th period since authorization was made. Hence, we assume, that

$$I_t^G = \sum_{n=0}^{N-1} \phi_n A_{t-n}^{IG}. \tag{73}$$

Since the investment has to be fully funded over the implementation period, $\sum_{n=0}^{N-1} \phi_n = 1$ holds.

2.5 Foreign Sector

Following the DSGE-model by Norges Bank, see Gerdrup et al. [2017], we model the foreign sector using a simple new-Keynesian exogenous block in order to capture the responses of the domestic economy to international shocks. In doing so we model interdependencies between the oil price, the world output (a proxy for world demand for the mainland's exports) as well as the demand for oil sector investments and capture thereby the important indirect effects of oil price changes on the domestic economy.

More specifically, we model foreign output as partly backward-looking and as having dynamic IS-curve features as well as responding negatively to oil price increases:

$$Y_t^* = \overline{Y}^* \left(\frac{Y_{t-1}^*}{\overline{Y}^*}\right)^{\rho_{Y^*}} \left(\frac{Y_{f,t}^*}{\overline{Y}_f^*}\right)^{1-\rho_{Y^*}} \left(\frac{P_{oil,t}}{\overline{P}_{oil}}\right)^{-\psi_{Y^*oil}} \epsilon_{Y^*,t} \tag{74}$$

where Y_t^* is the foreign (world) output, P_t^{oil} is the price of oil and $Y_{f,t}^*$ incorporates the standard IS curve dynamics:

$$Y_{f,t}^* = \overline{Y}_f^* \left(\frac{Y_{f,t+1}^*}{\overline{Y}_f^*}\right)^{\psi_{Yf^*}} \left(\frac{R_t^*}{\pi_{t+1}^*} / \frac{\overline{R}^*}{\overline{\pi}^*}\right)^{-\psi_{Y_f R^*}}.$$

Similarly, foreign inflation is partly backward-looking:

$$\pi_t^* = \overline{\pi}^* \left(\frac{\pi_{t-1}^*}{\overline{\pi}^*}\right)^{\rho_{\pi^*}} \left(\frac{\pi_{f,t}^*}{\overline{\pi}_f^*}\right)^{1-\rho_{\pi^*}} \left(\frac{P_{oil,t}}{\overline{P}_{oil}}\right)^{\psi_{\pi^*oil}} \epsilon_{\pi^*,t} \tag{75}$$

where π_t^* is the foreign (world) inflation and π_f^* incorporates the standard forward-looking Phillips-curve dynamics:

$$\pi_{f,t}^* = \overline{\pi}_f^* \left(\frac{\pi_{f,t+1}^*}{\overline{\pi}_f^*}\right)^{\psi_{\pi_f^*}} \left(\frac{Y_t^*}{\overline{Y}^*}\right)^{\psi_{\pi_f Y^*}}.$$

The foreign monetary policy is run by a standard Taylor rule where interest rate responds to the contemporaneous world inflation and output:

$$R_t^* = \overline{R}^* \left(\frac{R_{t-1}^*}{\overline{R}^*}\right)^{\psi_{r^*}} \left(\left(\frac{\pi_t^*}{\overline{\pi}}\right)^{\psi_\pi^*} \left(\frac{Y_t^*}{\overline{Y}^*}\right)^{\psi_y^*}\right)^{1-\psi_{r^*}} \exp(\epsilon_r^*). \tag{76}$$

The international oil price dynamic is determined by the world demand and its own lag:

$$P_{oil,t} = \overline{P}_{oil} \left(\frac{P_{oil,t-1}}{\overline{P}^*_{oil}} \right)^{\rho_{oil}} \left(\frac{Y^*_t}{\overline{Y}^*} \right)^{\psi_{oilY^*}} \epsilon_{oil,t} \tag{77}$$

While we do not model the oil production sector of Norway explicitly we add to the model an exogenous demand for investment goods by the oil production sector, which is also a function of the oil price, and given by

$$\frac{I^{OIL}_t}{\overline{I^{OIL}}} = \left(\frac{I^{OIL}_{t-1}}{\overline{I^{OIL}}} \right)^{\rho_{OILI}} \left(\frac{P_{oil,t}}{\overline{P}^*_{oil}} \right)^{\psi_{oili}} \exp(\epsilon_{OILI,t}) \tag{78}$$

where $\overline{I^{OIL}}$ is the steady-state oil sector investment demand and $\epsilon_{OILI,t}$ is an exogenous shock. The oil sector purchases of domestically produced investment goods enter the balance of payments since they represent purchases payed for from outside of the mainland economy.

2.6 Macroeconomic relationships

While we have already presented output volumes at the firm level, we will introduce output at the aggregate level in this section. Furthermore we discuss the balance of payments in the economy as well as the aggregate market clearing.

2.6.1 Home production and GDP

The total volume of domestic production, $Y_{H,t}$, which is undertaken by intermediate level firms, consists of domestically produced consumption, $C_{H,t}$, domestically produced investment good, $I_{H,t}$, government purchases of goods and services C^G_t and exports, X_t. Hence,

$$Y_{H,t} = \underbrace{C_{H,t} + I_{H,t} + C^G_t}_{Y^D_{H,t}} + X_t, \tag{79}$$

where $Y^D_{H,t}$ denotes domestically sold home production.[21] Note, that the final good sector is not contributing any value-added but only bundles domestic and imported goods with no use of capital and labor and and thus can be left out from the definition of total domestic production.

The GDP is defined as the sum of domestic output, the government wage bill , public capital depreciation and inventory changes, INV_t, i.e.

$$Y_t = Y_{H,t} + \frac{(1+\tau^{SS,F}_t) w^G_t N^G_t}{P_{h,t}} + \frac{P^i_t \delta_{KG} K_{G,t}}{P_{h,t}} + INV_t \tag{80}$$

where INV_t is given by an exogenous process. The wage bill as well as the public capital depreciation are divided by the relative price of the home good to translate their values which are given in CPI-terms in units of the domestic good. Note, that the factors preceding N^G_t and $K_{G,t}$ are held constant at their steady-state value following the national accounts convention that government employment and capital depreciation are to be valued at base prices. As a consequence, only volume changes (i.e. changes in public employment or the public capital stock) affect the government wage bill and public capital depreciation in the GDP definition.

[21] Note, that since C^G_t is assumed to have full home bias it is not a component of C_t, and thus not included in $C_{H,t}$ whereas public investments I^G_t are a component of I_t and thus $I_{H,t}$ since they exhibit the same import share as other investments.

We also define GDP in CPI units, Y_t^{CPI}, given by the sum of the components of Y_t expressed in CPI units. Note, that a different relative price is applied to exports due to local currency pricing:

$$Y_t^{CPI} = P_{h,t}Y_{H,t}^D + Q_t P_{x,t}^* X_t + (1+\tau_t^{SS,F})\overline{w_t^G} N_t^G + \overline{P_t^i}\delta_{KG}K_{G,t} + P_{h,t}INV_t. \tag{81}$$

The relative price of GDP, defined as $P_t^{gdp} = \frac{Y_t^{CPI}}{Y_t}$, equals the weighted sum of relative prices of the GDP components, i.e. $P_{h,t}\frac{Y_t - X_t}{Y_t} + Q_t P_{x,t}^* \frac{X_t}{Y_t}$.

2.6.2 Balance of Payments

We first introduce a measure of net exports in the economy. Since exports, X_t, and imports, IM_t, are measured in different units of account, we apply the respective relative prices onto them to arrive at CPI units to be able to measure the difference in values, i.e.

$$NX_t = Q_t P_{x,t}^* X_t - P_{f,t} IM_t. \tag{82}$$

We now can introduce the balance of payments that represents payments that occur across the border of the model economy. For example a negative value of net exports can imply that the domestic economy is reducing the level of foreign bonds it holds because it has to pay for the import excess in the foreign currency. In general, all payments entering the economy have to equal payments leaving it. Hence it holds, that

$$\begin{aligned} &\underbrace{NX_t}_{\text{net exports}} + \underbrace{P_t^i I_t^{OIL}}_{\text{Oil sector investment}} + \underbrace{OILR_t}_{\text{Oil fund withdrawals}} \\ = &\underbrace{\frac{e_t P_t^*}{P_t} B_t}_{\text{foreign bond value at time } t} - \underbrace{\frac{e_t P_{t-1}^*}{P_t} B_{t-1} R_{t-1}^* \phi_{t-1}(A_{t-1})}_{\text{foreign bond value at time } t-1}. \end{aligned} \tag{83}$$

The left hand side denotes payments from outside to the domestic economy, consisting of payments equivalent to the value of net exports (possibly negative), public assets withdrawals and payments equivalent to the value of oil sector investments. Note, that the latter represent goods that are exported to outside of the model domestic economy which is calibrated to represent the mainland economy which implies that the oil production sector is viewed as an foreign entity outside of the borders of the mainland economy.[22] On the right hand side of the equation we observe the net change in foreign bonds which represent payments originating from the domestic economy.

2.6.3 Aggregate Market Clearing

Using the balance of payments equation (83), the government budget equation (66), the Rule-of-Thumb budget equation (23), the profit functions (62) as well as the definition of

[22] Note, that this stands in contrast to official statistics on the balance of payments as the oil producing industry is in reality a part of the overall Norwegian economy. However, including the oil sector into the domestic economy is in conflict with our wish to only capture the mainland economy in our calibration and has further drawbacks. For example, when modeling the oil sector, its production and export would need to enter the balance of payments. Simultaneously, foreign bonds B would then need to also contain assets held within the oil fund. Due to the sheer size of the pension fund, the behaviour of the optimizing household would then be strongly dominated by its wealth held in foreign bonds and particularly through exchange rate movements. We consider the resulting model dynamics as highly unlikely. The decision to only model the mainland economy as the domestic economy, however, allows us to treat the government pension fund as fully exogenous and thus invisible to the household. Instead, the household only reacts to the level of fund withdrawals. This strikes us as the more realistic assumption for a medium-term model such as this.

GDP in (80) and inserting it into the budget constraint (8) we obtain the aggregate market clearing condition

$$
\begin{aligned}
Y_t^{CPI} &= C_t + P_{h,t}C_t^G + P_t^i(I_t + I_t^G + I_t^{OIL}) + NX_t + \\
&\quad (1+\tau_t^{SS,F})w_t^G N_t^G + P_t^i \delta_{KG} K_{G,t} + P_{h,t} INV_t + AC_t
\end{aligned} \tag{84}
$$

where $AC_t = AC_{H,t} + AC_{X,t} + \gamma_t$ are adjustment costs. The derivation of the good market clearing can be found in the appendix section 6.8. This condition differs from the GDP definition in (81) insofar as the latter expresses GDP as the sum of domestic production, whereas equation (84) expresses GDP as the sum of total expenditures. Hence, the derived relationship captured by the aggregate market clearing shows that on aggregate the production of resources equals its consumption.

3 Calibration

The model is calibrated to the Norwegian mainland economy in a two-step process. In the first stage a subset of the parameters and exogenous variables are chosen such that the deterministic steady state replicates a number of long-run moments from Norwegian national accounts.[23] The long-run targets together with their empirical counterparts are displayed in Table 2. As a test of the model, we also display the implied steady state of a range of non-targeted variables together with their empirical counterparts. In the second stage, the remaining parameters are chosen to yield impulse responses for a monetary policy and technology shock that are quantitatively comparable to those from NEMO IV, see Gerdrup et al. [2017], and qualitatively comparable to fiscal policy models developed for other countries. A summary of all structural parameters can be found in Table 3.

3.1 Steady-state targets

The gross inflation rate in Norway (π) is calibrated to two percent annually, consistent with Norges Banks inflation target. For convenience, we set inflation abroad to the same value. The discount factor (β) is set at 0.998, yielding a steady-state nominal interest rate of 3.8 percent per annum, see Appendix 6.7 for more details on how to derive the steady state. We set the relative price of domestic production (P^h) and investment (P^f) to 1 in steady state for simplicity and set the α_C to 0.39 and α_I to 0.47 to match the steady-state import shares of consumption and investment. We set the deprecation rate of private capital (δ) to 0.017 (approximately 7 percent per annum) to match the observed investment to GDP ratio in the data. The depreciation rate of public capital (δ_{KG}) is set to 0.020 (approximately 8.2 percent per annum) to match the empirical public investment to GDP ratio. Since in our model the public investment to GDP ratio must equal depreciated public capital in the steady state, we can not match both empirical moments simultaneously. That is why we overestimate government capital depreciation as a GDP component, which is, however, exactly compensated by an underestimation of the inventory component. Taking the capital share in the production function and the elasticity of substitution between private and public capital as given (see below), we set the parameter determining the share of public capital in the capital composite to 0.38 to match the empirical private capital to GDP ratio. The private (N^P) and public (N^G) sector employment to population ratios are set to 0.48 and 0.20 to match their empirical counterparts. The labor disutility scaling parameter (θ) is set to 2.31 to match the labor force participation rate. The elasticity of substitution between differentiated labor inputs (ϵ_W) is set to 7.59 enabling us to match

[23] The national accounts data can be downloaded on www.ssb.no. Further details are available on request.

Table 2: Overview on steady-state calibration

Description	Notes	Model	Empirics	Target
Consumption	GDP component, as % of GDP	51.68 %	51.73 %	51.73 %
Government purchases	GDP component, as % of GDP	6.66 %	6.66 %	6.66 %
Government wage bill	GDP component, as % of GDP	16.88 %	16.88 %	16.88 %
Government capital depreciation	GDP component, as % of GDP	5.58 %	3.83 %	-
Government investment	GDP component, as % of GDP	5.58 %	5.58 %	5.58 %
Private investment	GDP component, as % of GDP	15.21 %	15.21 %	15.21 %
Oil sector investment	GDP component, as % of GDP	7.30 %	7.30 %	7.30 %
Imports	GDP component, as % of GDP	33.38 %	33.40 %	33.40 %
Exports	GDP component, as % of GDP	22.40 %	22.40 %	22.40 %
Net Exports	GDP component, as % of GDP	-10.98 %	-11.00 %	-11.00 %
Inventory	GDP component, as % of GDP	2.09 %	3.84 %	-
Private capital stock	Stock, as % of GDP	920.80 %	920.80 %	920.80 %
Public capital stock	Stock, as % of GDP	277.60 %	277.60 %	277.60 %
Net foreign assets	Stock, as % of GDP	0.00 %	-50.44 %	0.00 %
Government Debt	Stock, as % of GDP	158.60 %	158.60 %	158.60 %
Unemployment benefits	Gov. budget component, as % of GDP	0.56 %	0.56 %	0.56 %
Transfers	Gov. budget component, as % of GDP	24.38 %	24.38 %	24.38 %
Public assets withdrawals	Gov. budget component, as % of GDP	3.68 %	5.84 %	-
Lump-sum taxation	Gov. budget component, as % of GDP	6.73 %	-	-
Transfers to Rot HH	Gov. budget component, as % of GDP	26.64 %	-	-
Transfers to Ric. HH	Gov. budget component, as % of GDP	23.41 %	-	-
Consumption tax rate	Tax rate	18.89 %	18.89 %	18.89 %
Ordinary income tax rate	Tax rate	25.40 %	25.40 %	25.40 %
Bracket tax rate	Tax rate	2.07 %	2.07 %	2.07 %
Social security HH rate	Tax rate	8.11 %	8.11 %	8.11 %
Social security firms rate	Tax rate	14.94 %	14.94 %	14.94 %
Corporate tax rate	Tax rate	27.23 %	27.23 %	27.23 %
Total employment rate	Employment	68.17 %	68.17 %	68.17 %
Private sector employment rate	Employment	20.42 %	20.42 %	20.42 %
Public sector employment rate	Employment	47.75 %	47.75 %	47.75 %
Unemployment rate	Employment	3.89 %	3.90 %	-
Labor force participation rate	Employment	70.93 %	70.93 %	70.93 %
Labor income share	Income Share	59.90 %	47.14 %	-

the unemployment rate in the data. The size of the foreign economy is set to match the historical export-to-GDP ratio. The capital share in production (α) and the share of public capital in overall capital are set to match the corresponding steady-state capital ratios, while the private and public capital deprecation rates (δ and δ_{KG}) are set to match the ratios of private non-oil investments and public investment to GDP. Oil investments, which are exogenous in the model, are calibrated directly. The level of public asset withdrawals is set, such that the net foreign asset position evaluates to zero, given the other components of the mainland economy balance of payments from (83) which are exactly matched. By choosing net foreign assets to be zero we follow standard practice of most DSGE models and thereby avoid the unwanted wealth effects on household balance sheets through exchange rate fluctuations.[24]

3.2 Other parameter values

The intertemporal elasticity of substitution (σ) is set close to 1 to approximate the log in-period utility function for consumption used in NEMO IV and much of the literature. A higher value would reduce the responsiveness of private consumption relative to NEMO IV. The labor disutility parameter (ψ) has a big impact on the effect of a change in

[24]The alternative approach to calibrate public asset withdrawals exactly and in turn accept a non-zero NFA has the drawback of exactly those wealth effects.

Table 3: Overview of parameter values used

Variable	Description	Value
α_C	Import share of consumption	0.39
β	Discount Factor	0.998
χ_A	risk premium parameter for NFA (A)	0.05
χ_e	risk premium parameter for dER	0.05
σ	intertemporal elasticity of substitution	1.01
ψ	inverse Frisch elasticity	3
ϵ_W	Elasticity of substitution across differentiated labor inputs	7.59
θ	Relative weight of labor in utility	2.31
h	Habit persistence parameter	0.4
η	Import vs domestic elasticity	0.5
η_x	Import vs domestic elasticity in foreign economy	2.5
ϵ_h	Elasticity of substitution among home goods	6
ϵ_f	Elasticity of substitution among foreign goods	6
ϵ_x	Elasticity of substitution among exported goods	6
χ_W	Adjustment cost parameter for wages	100
χ_{aW}	Weight on indexation in wage adjustments	0.5
χ_h	Adjustment cost parameter for domestically produced goods	200
χ_f	Adjustment cost parameter for imported goods	200
χ_x	Adjustment cost parameter for exported goods	100
χ_a	Weight on indexation in price adjustments	0.5
α_I	Import share of investment good	0.47
η_I	Import vs domestic elasticity for Investment	0.5
ω	Share of RoT HH	0.3
α	Capital elasticity in Cobb Douglas production function	0.4
α_K	share of public capital in the capital composite	0.38
η_K	elasticity of substitution between private and public capital	0.84
γ_{u2}	Parameter for Adj cost to u	0.1
δ	Private capital depreciation rate	0.017
$\delta_K G$	Government capital depreciation rate	0.020
χ_K	Parameter governing Adj cost to investment	2.5

the taxation of labour income. It is set to 3 as in NEMO III. Note that due to the presence of unemployment, unemployment benefits, and public employment the inverse of this parameter is no longer equal to the Frisch elasticity. A higher value could therefore be warranted in light of evidence from KVARTS regarding the impact of a change in the taxation of labour income. The habit persistence parameter (h) is set to 0.9, comparable to the value in NEMO IV where it is 0.835. The share of rule of thumb households (ω) is currently set at 0.3, which is within the range typically used in the literature, but may be adjusted based on evidence regarding the response of private consumption to government consumption in KVARTS. The elasticity of substitution between domestic and imported goods is set at 0.5 for both consumption (η) and investment goods (η_I). This is identical to the value used in NEMO IV, but below that estimated by Naug on Norwegian data (2002).[25] Setting the value to 1.5 as in Naug (ibid) would move the response of the real exchange rate to a technology shock away from what we observe in NEMO IV. For simplicity, we keep the same elasticity for both consumption and investment goods, though one could argue

[25] As noted by Brubakk et al. (2006), however, this study is based on data for the industrial sector where would one expect the value to be higher than for the economy-wide average.

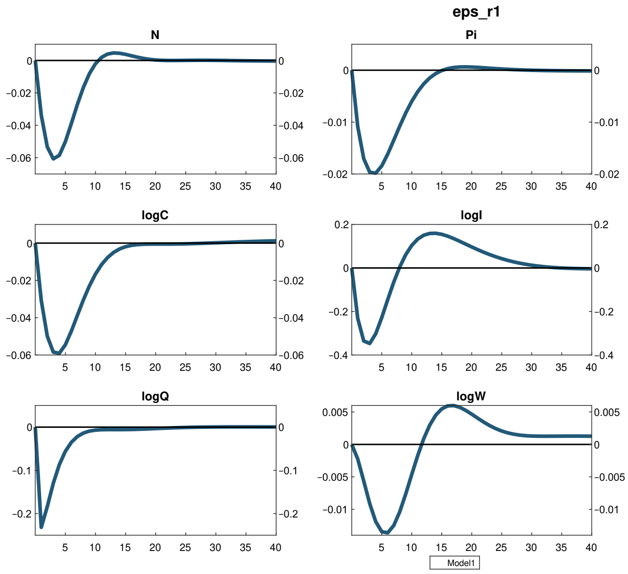
that a higher value for consumption goods may be warranted. The corresponding value for the foreign economy (η_x) is set at 2.5. Reducing this to 0.5 as in NEMO IV results in counterintuitive results for a technology shock. The elasticity of substitution between differentiated intermediate home goods can be related to the degree of competition in the domestic economy given that $\epsilon/(\epsilon - 1)$ can be interpreted as the price markup. We set the value to 6 for both domestic (ϵ_h), imported (ϵ_f), and exported goods (ϵ_x), consistent with a markup of 20 percent. This is slightly higher than the values estimated in NEMO IV, but allows our model to replicate the decline in real wages following an increase in interest rates from that model. The risk premium parameters in the model have a significant bearing on the response of the exchange rate, interest rates, and inflation in the model, and thereby also on investment. We set χ_A to 0.05, which is within the range of estimates in the empirical literature. A lower χ_A would increase the sensitivity of the real exchange rate following a technology shock relative to that in NEMO IV. We set χ_e to 0.05, which is significantly below the value estimated by Adolfsen et al. (2008). A higher χ_e would reduce the sensitivity of the real exchange rate to a monetary policy shock relative to what is in NEMO IV. It is worth noting that NEMO IV and most fiscal policy models set χ_e to zero such that the risk premium is solely a function of the net foreign asset position. The parameters capturing the cost of changing domestic (χ_h) and imported (χ_f) price are set to 200. The cost of changing the price of exports goods (χ_x) is set at 100. These are close to the values used in NEMO IV. The adjustment cost parameter for wages (χ_W) is set at 100. Increasing this would reduce the sensitivity of the real wage to movements in the interest rate relative to NEMO IV. The parameters determining the degree of backward indexation of prices (χ_a) and wages (χ_{aW}) are important for generating the hump-shaped response of inflation and wages that is observed empirically. We set both parameters at 0.8 which entails partial indexation. Full backward indexation (i.e. a value of 1) as in NEMO IV results in cycles that make the impulse responses difficult to interpret. The adjustment cost parameter for investment (χ_K) is set at 2.5 to match the decline in investment following an increase in interest rates from NEMO IV. We set the capital share in the production function (α) to 0.4. This is below the value in NEMO IV but above the value typically used in the empirical literature. A higher value reverses the sign of the investment response to a change in labor taxes. Simulations from KVARTS may therefore be able to shed more light on the value of this parameter. The elasticity of substitution between private and public capital (η_K) at 0.84 to match the estimate in Coenen et al. (2013). The parameter capturing the cost of changing the capital utilization rate ($\gamma_{u,2}$) is set at 0.1. The parameters of the monetary policy rule in the model correspond to those in NEMO IV.

4 Preliminary Results

4.1 Stochastic simulations

We first demonstrate the model's dynamics by performing stochastic simulations of standard aggregate shocks to our model economy. In particular, we consider a temporary shock to monetary policy involving a 10 basis point increase in the nominal interest rate as well as a temporary total factor productivity shock. We compare the impulse responses generated by our model to those arising from NEMO IV and documented in Gerdrup et al. [2017]. Note, that these simulations are presented with the main intention of providing a comparison to NEMO rather than a use-case of possible utilization of the model, which will follow in the next section.

Figure 4: Temporary increase in the nominal interest rate; Variables shown are employment, inflation, nominal interest rate (first row), consumption, investments and imports (second row), real exchange rate, real wage and GDP (last row)



4.1.1 Temporary increase in the nominal interest rate

Figure 4 shows a 10 percentage point (quarterly) increase in the nominal interest rate.[26] Due to nominal rigidities, higher nominal interest rates are accompanied by an increase in the real interest rate (not shown). This lowers both consumption and investment demand. Firms respond to lower demand by hiring fewer workers, offering lower real wages, and reducing output. The lower wages in turn reduce firms' marginal costs, resulting in lower inflation. The increase in interest rates results in an appreciation of the real exchange rate, which pushes down import prices and adds to the overall drop in inflation. The appreciation results in a shift towards imported goods but because of the decline in demand we nevertheless see a fall in imports. Exports (not shown) fall because of lower competitiveness, leaving net exports (not shown) broadly unchanged.

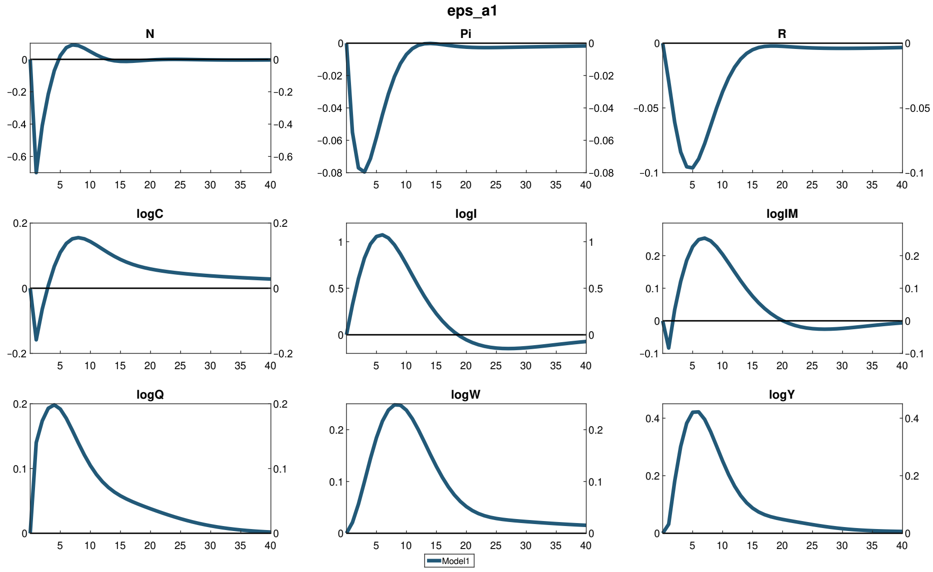
The effects of a monetary policy shock are comparable to those in NEMO IV and the empirical VAR literature. Relative to NEMO IV the response of real variables is somewhat more muted. This is not surprising given the lack of financial frictions in the current model. The results are also less inertial, with the peak on inflation occurring after 4-5 quarters compared to 7-8 quarters in NEMO IV. The (partly) backward-looking house price expectations in NEMO IV are likely part of the explanation for these differences. Another explanation is differences in the magnitude of real and nominal frictions (e.g. lower habit persistence) to compensate for the lack of amplification from financial frictions, as well the choice of less backward indexation to reduce cycles in the impulse response functions.

4.1.2 Temporary increase in total factor productivity

An increase in total factor productivity, see Figure 5 makes it possible for firms to produce the same amount of output with fewer inputs. This equates to a decline in their marginal

[26] The size of the shock is calibrated to facilitate comparisons with NEMO IV.

Figure 5: Temporary increase in total factor productivity; Variables shown are employment, inflation, nominal interest rate (first row), consumption, investments and imports (second row), real exchange rate, real wage and GDP (last row)



cost. Lower marginal costs makes it profitable to increase output. An increase in aggregate supply puts downward pressure on prices so inflation falls. To make inflation return to target the central bank needs to lower nominal interest rates sufficiently that real rates also fall. Inertia in the interest rate rule means that interest rates only fall gradually. The drop in the interest rate translates directly into an increase in investments.

Optimizing households experience increasing dividends from firms who have seen their profits increase because of declining costs, and due to valuation effects on domestic bond holdings (lower inflation). Overall consumption falls on impact due to declining consumption by rule of thumb households who experience a drop in wage income fall because of lower labour demand by firms who are now able to produce more with less inputs. The decline in labour demand manifests itself in lower employment and lower nominal wages. However, the amount of wage stickiness is sufficient to ensure that nominal wages decline less than the fall in prices. Therefore, real wages increase.

According to the standard UIP condition, lower nominal interest rates relative to the rest of the world produce an anticipated appreciation of the currency. This is typically accomplished in part by an instantaneous depreciation of the nominal exchange rate. In our model, however, the nominal exchange rate appreciates because lower domestic interest rates push households to increase their holdings of foreign assets, which reduces the risk premium in the economy. The appreciation of the nominal exchange rate puts downward pressure on import prices, so that overall inflation falls less than domestic inflation. The increase in the relative price of imports to domestic goods encourages a shift away from imports. This substitution effect is, however, outweighed by the increase in domestic demand. Imports therefore increase. Despite the stronger currency the real exchange rate depreciates as lower marginal costs encourages exporters to lower their prices. The increased competitiveness results in an unambiguous increase in exports that outweighs the rise in imports, so that net exports increase (not shown).

4.2 Deterministic simulations

In the following we present simulations in which a variable of interest is shifted permanently, such as the level of government purchases or public employment. The simulations illustrate possible use-cases for how the model can be utilized to build insights into the transmission channels and quantitative implications of fiscal policy.[27]

4.2.1 Permanent increase in government purchases

Figure 6: Permanent increase in government purchases; Variables shown are government purchases over GDP, GDP, employment, unemployment (first row), real wage, CPI inflation, nominal interest rate and real exchange rate (second row), consumption, exports, imports and investment (last row)

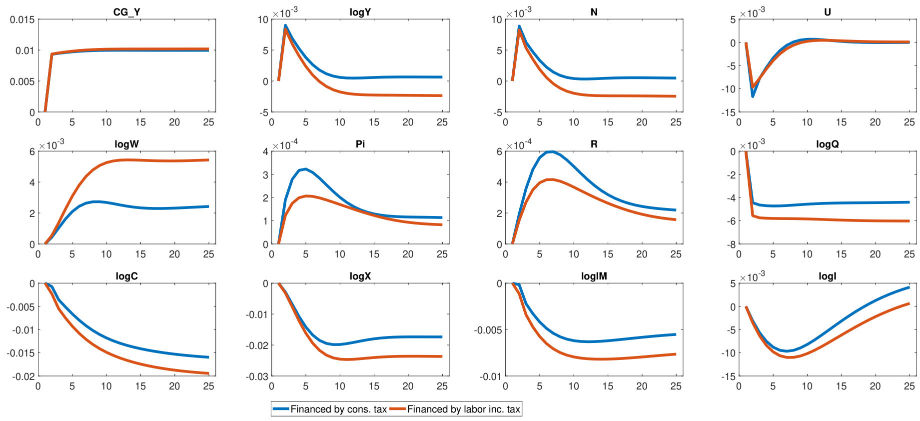


Figure 6 illustrates the impact of an increase in government purchases of the domestically produced good, i.e. CG, by 1 % of GDP. This increase in spending is simulated in two distinct runs, which differ by the type of financing used to keep the government budget balanced at each point in time such that government debt remains unchanged. The tax instruments considered are the consumption tax (blue line) and the bracket tax (red line).

The increase in CG is chosen such that the permanently higher level of expenditures is reached immediately in the first period of the shock, and identically so across experiments. The tax rates on consumption and labor (not shown) respond endogenously such that the government budget is balanced. The consumption tax increases by 2.1 pp in the long-run, while the bracket tax increases by 2.3 pp.

Due to price and wage stickiness the increase in public demand first results in a large expansion of employment in the short-run[28] (the employment rate increases by 1 pp) and a simultaneous fall in the unemployment rate. After a while, however, the increase in demand for the home good induces an increase in prices and inflation as well as a higher wage for the household. The increase in wages is higher when the expansion is financed by the tax on labor (the bracket tax) as the reduced after-tax wage induces the household

[27]The reasons permanent shifts are simulated deterministically, i.e. with perfect foresight, is that the solution method underlying stochastic simulations require shocks to be temporary allowing the economy to return to the steady state. This implies, however, that we can study temporary changes under stochasticity, too.

[28]Note, that we are working on making the response of employment more sluggish by introducing costs to changing employment on the side of employers and/or employees.

members to set a higher wage.

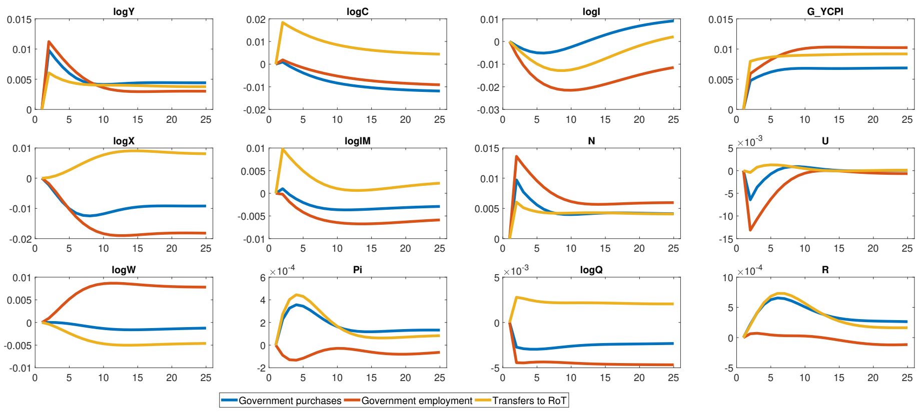
Due to the full home-bias of government purchases and the increase in the nominal interest rate (in line with the Taylor rule) the real exchange rate appreciates. As a consequence exports falls. Consumption declines in both scenarios due to the immediate effect of taxation lowering available income and the internalization of the permanently higher tax burden (only among optimizers). Financing by the bracket tax induces the households to reduce labor-supply in the long-run, reducing overall production and thus increasing the negative effect on consumption. For the same reason inflation is weaker under labor taxation. Imports directly follow the consumption response.

While inflation and the interest rate return to the initial steady state in the long-run (not shown), the price on the home good relative to the foreign good has permanently increased owing to a shift in demand away from foreign goods, explaining the permanent shift in the real exchange rate. The permanent shift away to domestic goods is larger under labor tax financing, which explains why appreciation is stronger in this scenario.

The increase in the real interest rate translates into lower investment in the short to medium run as the return on capital falls relative to the return on bonds. It remains at a lower level for the higher income tax due to permanently lower output.

4.2.2 Permanent increase in government expenditure components

Figure 7: Permanent increase in government expenditure components - Variables shown are GDP, consumption, investment, government expenditure over GDP (first row), exports, imports, employment and unemployment rate (second row), real wage, inflation, real exchange rate, nominal interest rate (last row)



In this section we consider an lump-sum tax financed expansion of government expenditures by 1 % of GDP achieved by an increase of public purchases (blue line), by an expansion of government employment (red line) and an increase in the transfers directed at Rule-of-Thumb households (yellow).[29]

[29]Note, that the size of the increase in these components is based on the initial values of the economy. For example, we increase government employment such, that the wage bill evaluated at the steady-state price increases by 1% of GDP. Since the economy will dynamically respond, including the wage rate itself, the actual increase in expenditure might differ from the targeted 1%. This approach, however, ensures comparability of the three experiments since in either case we assume an 1% increase in expenditures ex-ante.

The three expenditure shock have quite different effects on the economy. The increase in government purchases is a pure increase in aggregate demand, whilst the government employment shock affects mainly the labor market and household's income and the transfer shock is a redistribution of income to rule of thumb households since lump-sum taxes are levied only on optimizers.

All three shocks result in an immediate increase in aggregate employment and a fall in unemployment. In case of government purchases and the transfer shift this is due to an demand increase (either direct or induced by an increase in consumption of rule of thumb agents) and an increase in labor supply from optimizing households who pay higher taxes. This increase in labor supply also works to lower wages in these two scenarios and to increase in production. The increased demand also translates into higher inflation and nominal interest rates. The full home bias of government purchases leads to a permanent appreciation of the real exchange rate, while in the case of transfers demand shifts toward more import-intensive goods (consumption rather than investment) which depreciates the real exchange rate.

In the government employment scenario, however, the positive wage effect due to higher total labor demand from the private and public sector dominates and wages increase. Due to higher costs of labor private employment is partly crowded out, and domestic production falls (not shown), implying lower inflation. However, due to the increased wage bill, GDP nevertheless increases.

5 Summary and future work

In this paper we have presented a model of the Norwegian economy suitable for fiscal policy analysis. Going beyond most standard DSGE models, our framework features a rich government sector involving not only a detailed structure of government revenue instruments (taxes on households and firms, use of public savings) but also of public expenditures (transfers, investments, public wages and purchases). The model thus allows us to analyze the macroeconomic transmission channels of a variety of fiscal policy instruments.

Going forward, the project will focus on increasing the realism of the current model. Possible extensions include adding a more realistic depiction of wage formation in the Norwegian economy, a non-tradable sector, modelling long-run trends in the economy consistent with Norwegian data, and exploring household heterogeneity and alternative ways of modeling expectations formation.

This work is part of a ongoing project at the Ministry of Finance. The Ministry of Finances's Advisory Panel on Macroeconomic Models and Methods is regulary updated on progress with the project. This document will be presented to the Advisory Panel and subsequently published.

6 Appendix

6.1 First-order condition of the optimizing household

1. $\frac{\partial L}{\partial D_t^o} = 0$ yields

$$\begin{aligned} 0 &= \beta^{t+1} E_t(\lambda_{t+1}/\pi_{t+1})(1 + (R_t - 1)(1 - \tau_{t+1}^{OI})) - \beta^t \lambda_t \\ \Leftrightarrow \lambda_t &= \beta(1 + (R_t - 1)(1 - \tau_{t+1}^{OI}))E_t(\frac{\lambda_{t+1}}{\pi_{t+1}}) \end{aligned} \tag{85}$$

2. $\frac{\partial L}{\partial B_t^o} = 0$ yields

$$\begin{aligned} 0 &= \beta^{t+1} E_t(\lambda_{t+1} e_{t+1} P_t^*/P_{t+1})(1 + (R_t^* \phi_t(A_t) - \frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t})(1 - \tau_{t+1}^{OI})) - \beta^t \lambda_t Q_t \\ \Leftrightarrow Q_t \lambda_t &= \beta(1 + (R_t^* \phi_t(A_t) - \frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t})(1 - \tau_{t+1}^{OI}))E_t(\lambda_{t+1} e_{t+1} P_t^*/P_{t+1}) \\ \Leftrightarrow \lambda_t &= \beta(1 + (R_t^* \phi_t(A_t) - \frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t})(1 - \tau_{t+1}^{OI}))E_t(\lambda_{t+1} e_{t+1} P_t^*/P_{t+1} P_t/(e_t P_t^*)) \\ \Leftrightarrow \lambda_t &= \beta(1 + (R_t^* \phi_t(A_t) - \frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t})(1 - \tau_{t+1}^{OI}))E_t(\frac{\lambda_{t+1} e_{t+1}}{\pi_{t+1} e_t}) \end{aligned} \tag{86}$$

Subtracting these first-order conditions, we obtain the UIP condition

$$E_t(\lambda_{t+1}/\pi_{t+1}[(1+(R_t-1)(1-\tau_{t+1}^{OI}))-(1+(R_t^*\phi_t(A_t)-\frac{e_t}{e_{t+1}}\frac{P_{t+1}}{P_t})(1-\tau_{t+1}^{OI}))\frac{e_{t+1}}{e_t}]) = 0 \tag{87}$$

Using the definitions of after-tax nominal gross returns on domestic and foreign bonds we arrive at the equation in the main text. A linearized version of equation (87) is given by (where we drop the expectation operator for convenience):

$$\begin{aligned} \bar{R}(1 - \bar{\tau}^{OI})(\widehat{R}_t - \widehat{\tau_{t+1}^{OI}}) &= -(1 - \bar{\tau}^{OI})\widehat{\tau_{t+1}^{OI}} + \Delta\widehat{e}_{t+1} + \bar{R}^*(1 - \bar{\tau}^{OI})(\widehat{R_t^*} + (1 - \chi_e)\Delta\widehat{e}_{t+1} \\ &- \chi_e \Delta\widehat{e}_t - \chi_a \widehat{A}_t + \chi_O \widehat{OILR_t} - \widehat{\tau_{t+1}^{OI}}) - (1 - \bar{\tau}^{OI})\widehat{\pi}_{t+1} + \bar{\pi}\widehat{\tau_{t+1}^{OI}}. \end{aligned}$$

Holding the tax on ordinary income constant, this simplifies to

$$\begin{aligned} \bar{R}(1 - \bar{\tau}^{OI})\widehat{R}_t &= \Delta\widehat{e}_{t+1} + \bar{R}^*(1 - \bar{\tau}^{OI})(\widehat{R_t^*} + (1 - \chi_e)\Delta\widehat{e}_{t+1} \\ &- \chi_e \Delta\widehat{e}_t - \chi_a \widehat{A}_t + \chi_O \widehat{OILR_t}) - (1 - \bar{\tau}^{OI})\widehat{\pi}_{t+1}. \end{aligned}$$

The linearized UIP condition in the main text completely abstracts from bond taxation, giving rise to the well-known textbook version of it.
3. $\frac{\partial L}{\partial C_t^o} = 0$ yields

$$\begin{aligned} 0 &= \epsilon_{g,t}(C_t^o - hC_{t-1}^o)^{-\sigma} \frac{1}{(1-h)^{-\sigma}} - \lambda_t(1 + \tau_t^C) \\ \lambda_t &= \frac{\epsilon_{g,t}(C_t^o - hC_{t-1}^o)^{-\sigma}}{(1 + \tau_t^C)(1-h)^{-\sigma}} \end{aligned} \tag{88}$$

4. $\frac{\partial L}{\partial W_t(i)} = 0$
Note that this optimization is done for each i, i.e. for each profession.

$$
\begin{aligned}
0 &= \frac{\partial L}{\partial W_t(i)} \\
0 &= -\beta^t \epsilon_{g,t}\theta \int_0^1 N_t^o(i)^\psi \frac{\partial N_t^{o,P}(i)}{\partial W_t(i)} di \\
&+\beta^t \lambda_t \frac{1}{P_t}(1-\tau_t^W) \int_0^1 N_t^{o,P}(i) + W_t(i) \frac{\partial N_t^{o,P}(i)}{\partial W_t(i)} di - \beta^t \lambda_t UB_t \int_0^1 \frac{\partial N_t^{o,P}(i)}{\partial W_t(i)} di \\
&-\beta^t \lambda_t \int_0^1 \chi_W \left(\frac{W_t(i)/W_{t-1}(i)}{[(W_{t-1}/W_{t-2})^{\chi_{aW}}]\, \overline{\pi}^{(1-\chi_{aW})}} - 1 \right) \left(\frac{w_t/W_{t-1}(i)}{[(W_{t-1}/W_{t-2})^{\chi_{aW}}]\, \overline{\pi}^{(1-\chi_{aW})}} \right) di \\
&-\beta^{t+1} \lambda_{t+1} \int_0^1 \chi_W \left(\frac{W_{t+1}(i)/W_t(i)}{[(W_t/W_{t-1})^{\chi_{aW}}]\, \overline{\pi}^{(1-\chi_{aW})}} - 1 \right) \left(\frac{W_{t+1}(i)/W_t(i)}{[(W_t/W_{t-1})^{\chi_{aW}}]\, \overline{\pi}^{(1-\chi_{aW})}} \right) \frac{w_{t+1}}{-W_t(i)} di \\
&+\beta^t \lambda_t^L \left[\frac{\partial N_t^{o,P}(i)}{\partial W_t(i)} \epsilon_{g,t}\theta (L_t^o(i))^\psi - \lambda_t \left((1-\tau_t^W) \frac{1}{P_t} (N_t^{o,P}(i) + W_t(i) \frac{\partial N_t^{o,P}(i)}{\partial W_t(i)}) - UB_t \frac{\partial N_t^{o,P}(i)}{\partial W_t(i)} \right) \right]
\end{aligned}
$$

From equation (12), it follows that $\frac{\partial N_t^{o,P}(i)}{\partial W_t(i)} = -\epsilon_W \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W - 1} \frac{N_t^{o,P}}{W_t}$. Since all professions i face an identical first-order condition, labor demand function as well as participation equation they will all choose the same wage level, such that $W_t(i) = W_t$. Thus, the labor demand as well as participation will be identical across all professions. Thus, we drop the (i)-index in the following. Furthermore, to simplify the following calculation, we define

$$
DACW_t = \chi_W \left(\frac{W_t/W_{t-1}}{[(W_{t-1}/W_{t-2})^{\chi_{aW}}]\, \overline{\pi}^{(1-\chi_{aW})}} - 1 \right) \left(\frac{W_t/W_{t-1}}{[(W_{t-1}/W_{t-2})^{\chi_{aW}}]\, \overline{\pi}^{(1-\chi_{aW})}} \right). \tag{89}
$$

We then obtain

$$
\begin{aligned}
0 \;=\; & -\beta^t \epsilon_{g,t}\theta (N_t^o)^\psi (-\epsilon_W) \frac{N_t^{o,P}}{W_t} + \beta^t \lambda_t \left[\frac{1}{P_t}(1-\tau_t^W)(N_t^{o,P} - \epsilon_W N_t^{o,P}) - UB_t(-\epsilon_W) \frac{N_t^{o,P}}{W_t} \right. \\
& \left. - \frac{1}{P_t} DACW_t \right] + \beta^{t+1} \lambda_{t+1} \left[\frac{1}{P_{t+1}} DACW_{t+1} \frac{W_{t+1}}{W_t} \right] \\
& + \beta^t \lambda_t^L \left[-\epsilon_W \frac{N_t^{o,P}}{W_t} \epsilon_{g,t}\theta (L_t^o)^\psi - \lambda_t \left((1-\tau_t^W) \frac{1}{P_t} (N_t^{o,P} - \epsilon_W N_t^{o,P}) + UB_t \epsilon_W \frac{N_t^{o,P}}{W_t} \right) \right],
\end{aligned} \tag{90}
$$

dividing by $\frac{\beta^t N_t^{o,P} \lambda_t}{P_t}$ gives

$$
\begin{aligned}
0 &= \frac{\epsilon_{g,t}\theta (N_t^o)^\psi \epsilon_W}{w_t \lambda_t} + (1-\tau_t^W)(1-\epsilon_W) + UB_t \epsilon_W \frac{1}{w_t} - \frac{DACW_t}{N_t^{o,P}} \\
&+\beta \frac{\lambda_{t+1}}{\lambda_t} DACW_{t+1} \frac{w_{t+1}}{w_t} \frac{1}{N_t^{o,P}} + \frac{\lambda_t^L}{\lambda_t} \left(-\epsilon_W \frac{1}{w_t} \epsilon_{g,t}\theta (L_t^o)^\psi \right) \\
&-\lambda_t^L \left((1-\tau_t^W)(1-\epsilon_W) + UB_t \epsilon_W \frac{1}{w_t} \right),
\end{aligned} \tag{91}
$$

which yields the relationship given in the main text.
5. $\frac{\partial L}{\partial L_t^o(i)} = 0$ yields

$$0 = \lambda_t UB_t \int_0^1 1di + \lambda_t^L \left\{ N_t^o(i)\epsilon_{g,t}\theta\psi(L_t^o(i))^{\psi-1} - \lambda_t UB_t \right\}$$

$$\lambda_t UB_t = \lambda_t^L \left\{ \lambda_t UB_t - N_t^o(i)\epsilon_{g,t}\theta\psi(L_t^o(i))^{\psi-1} \right\} \tag{92}$$

6. $\frac{\partial L}{\partial K_{t+1}^o} = 0$ yields

$$\begin{aligned} 0 &= -\beta^t \mu_t + \beta^{t+1} E_t(\lambda_{t+1}[(1-\tau_{t+1}^{OI})R_{t+1}^k u_{t+1} + \tau_{t+1}^{OI} P_{t+1}^i(\delta_0 + \Gamma(u_{t+1})) - \Gamma(u_{t+1})P_{t+1}^i] \\ &\quad + \mu_{t+1}(1-\delta_0)) \\ \mu_t &= \beta E_t(\lambda_{t+1}[(1-\tau_{t+1}^{OI})R_{t+1}^k u_{t+1} + \tau_{t+1}^{OI} P_{t+1}^i \delta_0 - (1-\tau_{t+1}^{OI})\Gamma(u_{t+1})P_{t+1}^i] \\ &\quad + \mu_{t+1}(1-\delta_0)) \end{aligned} \tag{93}$$

7. $\frac{\partial L}{\partial I_t^o} = 0$ yields

$$\begin{aligned} 0 &= -\lambda_t P_t^i + \mu_t \left[-\chi_K(\frac{I_t^o}{I_{t-1}^o} - 1) I_t^o / I_{t-1}^o + \left(1 - \frac{\chi_K}{2} (\frac{I_t^o}{I_{t-1}^o} - 1)^2 \right) \right] \\ &+ \beta \mu_{t+1}(-\chi_K)(\frac{I_{t+1}^o}{I_t^o} - 1) I_{t+1}^o \frac{-I_{t+1}^o}{(I_t^o)^2} \end{aligned} \tag{94}$$

8. $\frac{\partial L}{\partial u_t} = 0$ yields

$$\begin{aligned} 0 &= \lambda_t \beta^t ((1-\tau_t^{OI}) R_t^k K_t^o - (1-\tau_t^{OI})\dot{\Gamma}(u_t) P_t^i K_t^o) \\ R_t^k &= (\chi_{u,1} + \chi_{u,2}(u_t - 1)) P_t^i \end{aligned} \tag{95}$$

Note that this equation determines $\chi_{u,1}$ in the steady state, since $u = 1$ in that case. We are thus not free to chose $\chi_{u,1}$ and instead calculate $\chi_{u,1}$ from the steady-state value of R and P_t^i in our numerical implementation of the model

6.2 Final good sector cost minimization

Consumption: Cost minimization implies

$$\min_{C_{H,t}, C_{F,t}} P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$$

giving rise to the Langrangian

$$\mathcal{L} = P_{H,t} C_{H,t} - P_{F,t} C_{F,t} + P_t \left(C_t - \left[(1-\alpha_C)^{1/\eta_c} (C_{H,t})^{\frac{\eta_c - 1}{\eta_c}} + \alpha_C^{1/\eta_c} (C_{F,t})^{\frac{\eta_c - 1}{\eta_c}} \right]^{\eta_c/(\eta_c - 1)} \right).$$

Note, that the Lagrange multiplier is identified to be P_t since the marginal cost of final good production sector (which is the economic interpretation of the Lag. mult.) equals the final good price due to perfect competition.

1. $\frac{\partial \mathcal{L}}{\partial C_{H,t}} = 0$ implies

$$\begin{aligned} P_{H,t} &= P_t \frac{\eta_c}{\eta_c - 1} [...]^{\frac{\eta_c}{\eta_c - 1} - 1} (1-\alpha_C)^{1/\eta_c} \frac{\eta_c - 1}{\eta_c} (C_{H,t})^{\frac{\eta_c - 1}{\eta_c} - 1} \\ \Leftrightarrow \left(\frac{P_{H,t}}{P_t} \right) &= [...]^{\frac{1}{\eta_c - 1}} (1-\alpha_C)^{1/\eta_c} (C_{H,t})^{\frac{-1}{\eta_c}} \\ \Leftrightarrow \left(\frac{P_{H,t}}{P_t} \right)^{\eta_c} &= [...]^{\frac{\eta_c}{\eta_c - 1}} (1-\alpha_C)(C_{H,t})^{-1} \\ \Leftrightarrow C_{H,t} &= (1-\alpha_C)(P_{h,t})^{-\eta_c} C_t \end{aligned}$$

2. $\frac{\partial \mathcal{L}}{\partial C_{F,t}} = 0$ implies analogously

$$C_{F,t} = \alpha_C \left(P_{f,t}\right)^{-\eta_c} C_t$$

It then follows through the profit function of final good firm (using the fact that these are perfectly competitive), that

$$\begin{aligned}
P_t C_t &= P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \\
&= (1-\alpha_C)\left(\frac{(P_{H,t})^{1-\eta_c}}{P_t^{-\eta_c}}\right) C_t + \alpha_C \left(\frac{(P_{F,t})^{1-\eta_c}}{P_t^{-\eta_c}}\right) C_t \\
\Leftrightarrow P_t &= (\frac{1}{P_t})^{-\eta_c} \left((1-\alpha_C)\left(P_{H,t}\right)^{1-\eta_c} + \alpha_C \left(P_{F,t}\right)^{1-\eta_c}\right) \\
\Leftrightarrow P_t &= \left((1-\alpha_C)\left(P_{H,t}\right)^{1-\eta_c} + \alpha_C \left(P_{F,t}\right)^{1-\eta_c}\right)^{1/1-\eta_c} \\
\Leftrightarrow 1 &= (1-\alpha_C)(P_{h,t})^{1-\eta_c} + \alpha_C (P_{f,t})^{1-\eta_c}.
\end{aligned}$$

Investment: Cost minimization implies

$$\min_{I_{H,t}, I_{F,t}} P_{H,t} I_{H,t} + P_{F,t} I_{F,t}$$

Lagrange

$$\mathcal{L} = P_{H,t} I_{H,t} + P_{F,t} I_{F,t} + P_t^I \left(\widehat{I}_t - \left[(1-\alpha_I)^{1/\eta_I} (I_{H,t})^{\frac{\eta_I - 1}{\eta_I}} + \alpha_I^{1/\eta_I} (I_{F,t})^{\frac{\eta_I - 1}{\eta_I}}\right]^{\eta_I/(\eta_I - 1)}\right)$$

Note, that the Lagrange multiplier is identified to be P_t^I since the marginal cost of final good production sector (which is the economic interpretation of the Lag. mult.) equals the final good price due to perfect competition.
1. $\frac{\partial \mathcal{L}}{\partial I_{H,t}} = 0$ implies (analogously to calculation for C_t)

$$\begin{aligned}
I_{H,t} &= (1-\alpha_I)\left(\frac{P_{H,t}}{P_t^I}\right)^{-\eta_I} \widehat{I}_t \\
I_{H,t} &= (1-\alpha_I)\left(\frac{P_{H,t}}{P_t}\frac{P_t}{P_t^I}\right)^{-\eta_I} \widehat{I}_t \\
I_{H,t} &= (1-\alpha_I)\left(P_{h,t}\right)^{-\eta_I} \left(P_t^i\right)^{\eta_I} \widehat{I}_t
\end{aligned}$$

2. $\frac{\partial \mathcal{L}}{\partial I_{F,t}} = 0$ implies (analogously to calculation for C_t)

$$\begin{aligned}
I_{F,t} &= \alpha_I \left(\frac{P_{F,t}}{P_t^I}\right)^{-\eta_I} \widehat{I}_t \\
I_{F,t} &= \alpha_I \left(P_{f,t}\right)^{-\eta_I} \left(P_t^i\right)^{\eta_I} \widehat{I}_t
\end{aligned}$$

It then follows through the profit function of final good firm, that

$$\begin{aligned}
P_t^I \widehat{I}_t &= P_{H,t} I_{H,t} + P_{F,t} I_{F,t} \\
&= (1-\alpha_I)\left(\frac{P_{H,t}}{P_t^I}\right)^{-\eta_I} \widehat{I}_t + \alpha_I \left(\frac{P_{F,t}}{P_t^I}\right)^{-\eta_I} \widehat{I}_t \\
\Leftrightarrow P_t^I &= \left((1-\alpha_I)\left(P_{H,t}\right)^{1-\eta_I} + \alpha_I \left(P_{F,t}\right)^{1-\eta_I}\right)^{1/1-\eta_I}
\end{aligned}$$

6.3 Intermediate sector cost minimization

The optimization is solved using the following Lagrangian function

$$L = (1+\tau_t^{SS,F})w_t N_t^P(i) + u_t K_t(i) R_t^k - \phi_t(i)\left(\epsilon_{a,t}(\widetilde{K_t}(i))^{\alpha}(N_t^P(i))^{1-\alpha} - Y_{H,t}(i)\right)$$

where $\phi_t(i)$ is the Lagrange multiplier associated with the production constraint of firm i. Cost minimization then implies that

1. $\frac{\partial L}{\partial N_t^P} = 0$

$$\begin{aligned} 0 &= (1+\tau_t^{SSF})w_t - \phi_t(i)\frac{(1-\alpha)Y_{H,t}(i)}{N_t^P(i)} \\ \Leftrightarrow w_t &= \phi_t(i)\frac{(1-\alpha)Y_{H,t}(i)}{(1+\tau_t^{SSF})N_t^P(i)} \end{aligned} \tag{96}$$

2. $\frac{\partial L}{\partial K_{p,t}(i)} = 0$

$$\begin{aligned} 0 &= u_t R_t^k - \phi_t(i)\frac{\alpha Y_{H,t}(i)}{\widetilde{K_t}(i)}\frac{\partial \widetilde{K_t}(i)}{\partial K_t(i)} \\ \Leftrightarrow R_t^k &= \phi_t(i)\frac{\alpha Y_{H,t}(i)}{\widetilde{K_t}(i)}\frac{1}{u_t}\frac{\partial \widetilde{K_t}(i)}{\partial K_t(i)} \\ \Leftrightarrow R_t^k &= \phi_t(i)\frac{\alpha Y_{H,t}(i)}{\widetilde{K_t}(i)}\left((1-\alpha_K)\frac{\widetilde{K_t}(i)}{u_t K_t(i)}\right)^{\frac{1}{\eta_K}} \\ \Leftrightarrow R_t^k &= \phi_t(i)\frac{\alpha Y_{H,t}(i)}{u_t K_t(i)}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{\widetilde{K_t}(i)}{u_t K_t(i)}\right)^{\frac{1}{\eta_K}-1} \\ \Leftrightarrow R_t^k &= \phi_t(i)\frac{\alpha Y_{H,t}(i)}{u_t K_t(i)}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K_t}(i)}\right)^{\frac{\eta_K-1}{\eta_K}} \end{aligned} \tag{97}$$

where the second equality follows from

$$\begin{aligned} \frac{1}{u_t}\frac{\partial \widetilde{K_t}(i)}{\partial K_t(i)} &= \frac{1}{u_t}\Big[(1-\alpha_K)^{1/\eta_K}(u_t K_t(i))^{\frac{\eta_K-1}{\eta_K}} \\ &\quad +\alpha_K^{1/\eta_K}(K_{G,t})^{\frac{\eta_K-1}{\eta_K}}\Big]^{\eta_K/(\eta_K-1)-1}(1-\alpha_K)^{1/\eta_K}(u_t K_t(i))^{\frac{\eta_K-1}{\eta_K}-1}u_t \\ &= \widetilde{K_t}(i)^{\frac{\eta_K-1}{\eta_K}(\frac{\eta_K}{\eta_K-1}-1)}(1-\alpha_K)^{1/\eta_K}(u_t K_t(i))^{\frac{-1}{\eta_K}} \\ &= \left((1-\alpha_K)\frac{\widetilde{K_t}(i)}{u_t K_t(i)}\right)^{\frac{1}{\eta_K}}. \end{aligned}$$

Inserting equations (96) with (97) in the total cost expression, we derive

$$\begin{aligned} TC_t(i) &= (1+\tau_t^{SS,F})w_t N_t^P(i) + u_t K_t(i) R_t^k \\ &= \phi_t(i)\left[(1-\alpha) + \alpha(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K_t}(i)}\right)^{\frac{\eta_K-1}{\eta_K}}\right]Y_{H,t}(i). \end{aligned}$$

Marginal cost are then given by

$$MC_t(i) = \frac{\partial TC_t(i)}{\partial Y_{H,t}(i)} = \phi_t(i)\left[(1-\alpha) + \alpha(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}}\right]. \tag{98}$$

From dividing equations (96) with (97) and rearranging we obtain the input efficiency conditions given in the main text.
From equation (45) it then follows, that

$$\frac{Y_{H,t}(i)}{u_t K_t(i)} = \frac{Y_{H,t}(i)}{\widetilde{K}_t(i)}\frac{\widetilde{K}_t(i)}{u_t K_t(i)} = \epsilon_{a,t}\frac{\widetilde{K}_t(i)}{N_t^P(i)}^{\alpha-1}\frac{\widetilde{K}_t(i)}{u_t K_t(i)} \tag{99}$$

Inserting equation (47) into equation (99) we obtain an expression that we can together with equation (98) insert into equation (97) and derive

$$\begin{aligned}
& R_t^k = \phi_t(i)\frac{\alpha Y_{H,t}(i)}{u_t K_t(i)}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}} \\
\overset{(98)}{\Leftrightarrow}\quad & R_t^k\left[(1-\alpha) + \alpha(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}}\right] = \\
& MC_t(i)\frac{\alpha Y_{H,t}(i)}{u_t K_t(i)}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}} \\
\overset{(47),(99)}{\Leftrightarrow}\quad & R_t^k\left[(1-\alpha) + \alpha(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}}\right] = \\
& MC_t(i)\alpha\epsilon_{a,t}\left(\frac{(1+\tau_t^{SSF})w_t}{R_t^k}\frac{\alpha}{1-\alpha}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{-1}{\eta_K}}\right)^{\alpha-1} \\
& \times\frac{\widetilde{K}_t(i)}{u_t K_t(i)}(1-\alpha_K)^{\frac{1}{\eta_K}}\left(\frac{u_t K_t(i)}{\widetilde{K}_t(i)}\right)^{\frac{\eta_K-1}{\eta_K}}
\end{aligned}$$

Solving for marginal cost we obtain the expression from the main text.

6.4 Intermediate sector domestic price setting

The problem of the firm is

$$\max_{P_{H,t}(i)} E_t \sum_{j=0}^{\infty} \Delta_{t,t+j}(1-\tau_t^{\Pi,F})\left[(P_{h,t}(i) - MC_t)Y_{H,t}^D(i) - AC_{H,t}(i)\right]$$

where the stochastic discount factor is given by

$$\Delta_{t,t+j} = \beta^j\frac{\lambda_{t+j}}{\lambda_t}.$$

After-tax profit maximization then yields

$$
\begin{aligned}
0 \quad = \quad & \beta^t\lambda_t(1-\tau_t^{\Pi,F})\Bigg\{(1-\epsilon_h)\left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\epsilon_h} Y_{H,t}^D - MC_t(-\epsilon_h)\frac{(P_{h,t}(i))^{-\epsilon_h-1}}{(P_{h,t})^{-\epsilon_h}}Y_{H,t} \\
& -\chi_h P_{h,t}Y_{H,t}\left[\frac{\frac{P_{h,t}(i)}{P_{h,t-1}(i)}\pi_t}{\left(\frac{P_{h,t-1}}{P_{h,t-2}}\pi_{t-1}\right)^{\chi_a}\overline{\pi}^{1-\chi_a}}-1\right]\left[\frac{\pi_t}{\left(\frac{P_{h,t-1}}{P_{h,t-2}}\pi_{t-1}\right)^{\chi_a}\overline{\pi}^{1-\chi_a}P_{h,t-1}(i)}\right]\Bigg\} \\
& -\beta^{t+1}\lambda_{t+1}(1-\tau_{t+1}^{\Pi,F})\Bigg\{\chi_h P_{h,t+1}Y_{H,t+1}^D\left[\frac{\frac{P_{h,t+1}(i)}{P_{h,t}(i)}\pi_{t+1}}{\left(\frac{P_{h,t}}{P_{h,t-1}}\pi_t\right)^{\chi_a}\overline{\pi}^{1-\chi_a}}-1\right] \\
& \times\left[\frac{\pi_{t+1}P_{h,t+1}(i)}{\left(\frac{P_{h,t}}{P_{h,t-1}}\pi_t\right)^{\chi_a}\overline{\pi}^{1-\chi_a}(-1)(P_{h,t}(i))^2}\right]\Bigg\}
\end{aligned}
$$

Since all firms arrive at this same optimal pricing equation, we can drop the firm index (i) and simplify to

$$
DAC_{H,t} = (1-\epsilon_h)+\epsilon_h(\mu_t^h)^{-1}+\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{H,t+1}^D}{Y_{H,t}^D}\frac{P_{h,t+1}}{P_{h,t}}\frac{(1-\tau_{t+1}^{\Pi,F})}{(1-\tau_t^{\Pi,F})}DAC_{H,t+1} \tag{100}
$$

where

$$
DAC_{H,t} = \chi_h\left[\frac{\frac{P_{h,t}}{P_{h,t-1}}\pi_t}{\left(\frac{P_{h,t-1}}{P_{h,t-2}}\pi_{t-1}\right)^{\chi_a}\overline{\pi}^{1-\chi_a}}-1\right]\left[\frac{\pi_t P_{h,t}}{\left(\frac{P_{h,t-1}}{P_{h,t-2}}\pi_{t-1}\right)^{\chi_a}\overline{\pi}^{1-\chi_a}P_{h,t-1}}\right]. \tag{101}
$$

6.5 Intermediate sector export price setting

The optimization problem of the exporter is

$$
\max_{P_{x,t}^*(i)} E_t\sum_{j=0}^{\infty}\Delta_{t,t+j}(1-\tau_t^{\Pi,F})\left[(P_{x,t}^*(i)Q_t-MC_{x,t})X_t(i)-AC_{X,t}(i)\right].
$$

Optimal pricing of export yields

$$
\begin{aligned}
0 \quad = \quad & \beta^t\lambda_t(1-\tau_t^{\Pi,F})\Bigg\{Q_tX_t(i)+P_{x,t}^*(i)Q_t(-\epsilon_x)\left(\frac{P_{x,t}^*(i)}{P_{x,t}^*}\right)^{-\epsilon_x-1}\frac{X_t}{P_{x,t}^*}-MC_{x,t}(-\epsilon_x)\frac{(P_{x,t}^*(i))^{-\epsilon_x-1}}{(P_{x,t}^*)^{-\epsilon_x}}X_t \\
& -\chi_xX_t(i)P_{x,t}^*Q_t\left[\frac{\frac{P_{x,t}^*(i)}{P_{x,t-1}^*(i)}\pi_t^*}{\left(\frac{P_{x,t-1}^*}{P_{x,t-2}^*}\pi_{t-1}^*\right)^{\chi_a}(\overline{\pi}^*)^{1-\chi_a}}-1\right]\left[\frac{\pi_t^*}{\left(\frac{P_{x,t-1}^*}{P_{x,t-2}^*}\pi_{t-1}^*\right)^{\chi_a}(\overline{\pi}^*)^{1-\chi_a}P_{x,t-1}^*(i)}\right]\Bigg\} \\
& -\beta^{t+1}\lambda_{t+1}(1-\tau_{t+1}^{\Pi,F})\Bigg\{\chi_xX_{t+1}(i)P_{x,t+1}^*Q_{t+1}\left[\frac{\frac{P_{x,t+1}^*(i)}{P_{x,t}^*(i)}\pi_{t+1}^*}{\left(\frac{P_{x,t}^*}{P_{x,t-1}^*}\pi_t^*\right)^{\chi_a}(\overline{\pi}^*)^{1-\chi_a}}-1\right] \\
& \times\left[\frac{\pi_{t+1}^*P_{x,t+1}^*(i)}{\left(\frac{P_{x,t}^*}{P_{x,t-1}^*}\pi_t^*\right)^{\chi_a}(\overline{\pi}^*)^{1-\chi_a}(-1)(P_{x,t}^*(i))^2}\right]\Bigg\}
\end{aligned}
$$

Since all the firms have the same optimization problem, the optimum price for each firm will be $P_{x,t}^*(i) = P_{x,t}^*$. Then, it follows that

$$
\begin{aligned}
0 \;=\; & \beta^t \lambda_t (1-\tau_t^{\Pi,F}) \Bigg\{ (1-\epsilon_x) Q_t X_t - MC_{x,t}(-\epsilon_x)(P_{x,t}^*)^{-1} X_t \\
& -\chi_x X_t P_{x,t}^* Q_t \left[\frac{\frac{P_{x,t}^*}{P_{x,t-1}^*}\pi_t^*}{\left(\frac{P_{x,t-1}^*}{P_{x,t-2}^*}\pi_{t-1}^*\right)^{\chi_a} (\overline{\pi}^*)^{1-\chi_a}} - 1 \right] \left[\frac{\pi_t^*}{\left(\frac{P_{h,t-1}^*}{P_{h,t-2}^*}\pi_{t-1}^*\right)^{\chi_a} (\overline{\pi}^*)^{1-\chi_a} P_{x,t-1}^*} \right] \Bigg\} \\
& -\beta^{t+1} \lambda_{t+1} (1-\tau_{t+1}^{\Pi,F}) \Bigg\{ \chi_x X_{t+1} P_{x,t+1}^* Q_{t+1} \left[\frac{\frac{P_{x,t+1}^*}{P_{x,t}^*}\pi_{t+1}^*}{\left(\frac{P_{x,t}^*}{P_{x,t-1}^*}\pi_t^*\right)^{\chi_a} (\overline{\pi}^*)^{1-\chi_a}} - 1 \right] \\
& \times \left[\frac{\pi_{t+1}^* P_{x,t+1}^*}{\left(\frac{P_{x,t}^*}{P_{x,t-1}^*}\pi_t^*\right)^{\chi_a} (\overline{\pi}^*)^{1-\chi_a} (-1)(P_{x,t}^*)^2} \right] \Bigg\}
\end{aligned}
$$

dividing all terms by X_t, Q_t, λ_t and β^t can simplify above as

$$
DAC_{X,t} = (1-\epsilon_x) + \epsilon_x \frac{MC_t}{Q_t P_{x,t}^*} + \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{X_{t+1}}{X_t} \frac{Q_{t+1}}{Q_t} \frac{(1-\tau_{t+1}^{\Pi,F})}{(1-\tau_t^{\Pi,F})} \left(\frac{P_{x,t+1}^*}{P_{x,t}^*} \right) DAC_{X,t+1} \quad (102)
$$

where

$$
DAC_{X,t} = \chi_x \left[\frac{\frac{P_{x,t}^*}{P_{x,t-1}^*}\pi_t^*}{\left(\frac{P_{x,t-1}^*}{P_{x,t-2}^*}\pi_{t-1}^*\right)^{\chi_a} (\overline{\pi}^*)^{1-\chi_a}} - 1 \right] \left[\frac{\pi_t^* P_{x,t}^*}{\left(\frac{P_{h,t-1}^*}{P_{h,t-2}^*}\pi_{t-1}^*\right)^{\chi_a} (\overline{\pi}^*)^{1-\chi_a} P_{x,t-1}^*} \right]. \quad (103)
$$

We have set $MC_{x,t} = MC_t$ since it is the domestic firms who are exporting and the marginal cost is identical for the exported and domestically sold intermediate good.

6.6 Import sector price setting

The problem of the firm is

$$
\max_{P_{F,t}(i)} E_t \sum_{j=0}^{\infty} \Delta_{t,t+j} (1-\tau_t^{\Pi,F}) \left[(P_{f,t}(i) - Q_t) IM_t(i) - AC_{F,t}(i) \right].
$$

Profit maximization then yields

$$
\begin{aligned}
0 \;=\; & \beta^t \lambda_t (1-\tau_t^{\Pi,F}) \Bigg\{ (1-\epsilon_f) \left(\frac{P_{f,t}(i)}{P_{f,t}} \right)^{-\epsilon_f} IM_t - Q_t(-\epsilon_f) \frac{(P_{f,t}(i))^{-\epsilon_h - 1}}{(P_{f,t})^{-\epsilon_f}} IM_t \\
& -\chi_f P_{f,t} IM_t \left[\frac{\frac{P_{f,t}(i)}{P_{f,t-1}(i)}\pi_t}{\left(\frac{P_{f,t-1}}{P_{f,t-2}}\pi_{t-1}\right)^{\chi_a} \overline{\pi}^{1-\chi_a}} - 1 \right] \left[\frac{\pi_t}{\left(\frac{P_{f,t-1}}{P_{f,t-2}}\pi_{t-1}\right)^{\chi_a} \overline{\pi}^{1-\chi_a} P_{f,t-1}(i)} \right] \Bigg\} \\
& -\beta^{t+1} \lambda_{t+1} (1-\tau_{t+1}^{\Pi,F}) \Bigg\{ \chi_f P_{f,t+1} IM_{t+1} \left[\frac{\frac{P_{f,t+1}(i)}{P_{f,t}(i)}\pi_{t+1}}{\left(\frac{P_{f,t}}{P_{f,t-1}}\pi_t\right)^{\chi_a} \overline{\pi}^{1-\chi_a}} - 1 \right] \\
& \times \left[\frac{\pi_{t+1} P_{f,t+1}(i)}{\left(\frac{P_{f,t}}{P_{f,t-1}}\pi_t\right)^{\chi_a} \overline{\pi}^{1-\chi_a} (-1)(P_{f,t}(i))^2} \right] \Bigg\}
\end{aligned}
$$

Since all firms arrive at this same optimal pricing equation, we can drop the firm index (i) and simplify to

$$DAC_{F,t} = (1-\epsilon_f) + \epsilon_f Q_t (P_{f,t})^{-1} + \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{IM_{t+1}}{IM_t} \frac{P_{f,t+1}}{P_{f,t}} \frac{(1-\tau_{t+1}^{\Pi,F})}{(1-\tau_t^{\Pi,F})} DAC_{f,t+1} \tag{104}$$

where

$$DAC_{F,t} = \chi_f \left[\frac{\frac{P_{f,t}}{P_{f,t-1}}\pi_t}{\left(\frac{P_{f,t-1}}{P_{f,t-2}}\pi_{t-1}\right)^{\chi_a} \overline{\pi}^{1-\chi_a}} - 1 \right] \left[\frac{\pi_t P_{f,t}}{\left(\frac{P_{f,t-1}}{P_{f,t-2}}\pi_{t-1}\right)^{\chi_a} \overline{\pi}^{1-\chi_a} P_{f,t-1}} \right]. \tag{105}$$

6.7 Steady-state Solution and Calibration

In this section variables without a t-subscript denote the steady-state values of the corresponding endogenous variables of the model.

1. **Inflation:** We impose a steady state on domestic and foreign inflation

$$\pi = \pi_{SS} \tag{106}$$
$$\pi^* = \pi_{SS}^* \tag{107}$$

   where π_{SS} and π_{SS}^* can be freely chosen.

2. **Taxes:** Since the tax rates in the model can be pinned down by the data, we set the steady-state tax rates to these empirically determined values.

$$\tau^i = \tau_{SS}^i \tag{108}$$

   where $i \in \{C; OI; BT; SS, H; SS, F; \Pi, F\}$.

3. **Relative prices, exchange rate, markup:** Rearranging the steady-state version of equation (36), we obtain

$$P_h = \left(\frac{P_h C_H}{C} \frac{1}{1-\alpha_C} \right)^{1/(1-\eta_C)}$$

   where the value of $\frac{P_h C_H}{C}$ is taken from the data and α_C set to any value (between 0 and 1).[30] The very same approach is applied on equation (37), to derive P_f. Hence, P_h and P_f are used as instruments to match the empirical import share of consumption, for any value of η_C. Inserting equation (96) into (41) we obtain

$$\frac{P_h I_H}{\widehat{I}} = (1-\alpha_I)(P_h^{1-\eta_I}) \left((1-\alpha_I) P_h^{1-\eta_I} + \alpha_I P_f^{1-\eta_I} \right)^{\frac{\eta_I}{1-\eta_I}}$$

   Using numerical methods, we can find a α_I such that this equation holds given an empirically determined value for $\frac{P_h I_H}{\widehat{I}}$, the already determined P_h, P_f and any choice of η_I. Hence, we choose α_I to determine the import share of investment. It follows from the steady-state version of the optimal import pricing equation, (104), that

$$Q = P_f \frac{\epsilon_f - 1}{\epsilon_f}. \tag{109}$$

[30]Ideally, one sets $\alpha_C = \frac{P_h C_H}{C}$, which has the attractive consequence, that $P_h = 1$.

Using the optimal home good pricing equation, (100), we derive the steady-state price mark-up as

$$\mu^h = \frac{\epsilon_h}{\epsilon_h - 1}. \tag{110}$$

Using (49) we then obtain marginal costs as $MC = P^h/\mu^h$.

Using the optimal pricing decisions for exports from equation (102), we obtain

$$P_x = \frac{\epsilon_x}{\epsilon_x - 1}\frac{MC}{Q}.$$

4. **Interest rates:**

   Using (85) and (106) we obtain

$$R = \frac{\frac{\pi_{SS}}{\beta} - 1}{1 - \tau^{OI}} + 1 \tag{111}$$

   Using (86) and (107) we obtain

$$R^* = \frac{\frac{\pi^*_{SS}}{\beta} - 1}{1 - \tau^{OI}} + \pi^*_{SS} \tag{112}$$

   where we have used the fact that the risk premium $\phi(A) = 1$ in the steady state as follows from the definition of ϕ_t.

5. **Adjustment costs** It follows directly from (89), (15), (52), (101), (60) and (105), that

$$\gamma^u = DACW = \gamma^W = AC_H = DAC_H = AC_H = DAC_H = 0.$$

6. **Depreciation:** From the steady-state version of the capital accumulation equation (13), it follows, that

$$\delta = \frac{P^i I}{Y^{CPI}}\left(\frac{K}{Y^{CPI}}\right)^{-1}\frac{1}{P^i},$$

   where both $\frac{P^i I}{Y^{CPI}}$ and $\frac{K}{Y^{CPI}}$ can be determined empirically. Hence, we choose δ such that we obtain the correct investment share over GDP. The parameter choice to make sure $\frac{K}{Y^{CPI}}$ holds will be done further below.

7. **Rental rate of capital:** Using equation (93), we obtain

$$\begin{aligned}
\mu - \beta(1-\delta_0)\mu &= \beta\lambda((1-\tau^{OI})R^k u + \tau^{OI}\delta_0 P^i) \\
\mu - \beta(1-\delta_0)\mu &= \beta\mu\frac{1}{P^i}((1-\tau^{OI})R^k u + \tau^{OI}\delta_0 P^i) \\
R^k &= (\frac{P^i}{\beta}(1-\beta(1-\delta_0)) - \tau^{OI}\delta_0 P^i)/(1-\tau^{OI}),
\end{aligned}$$

   where we imposed that $u = 1$ in the steady state.

8. **Capital-to-output ratio:** Taking α as given, we can use (97) to obtain

$$\frac{P^i K}{Y_H}\frac{1}{P^i} = \frac{1}{R^k}\alpha\phi(1-\alpha_K)^{1/\eta_K}\left(\frac{K}{\widetilde{K}}\right)^{\frac{\eta_K-1}{\eta_K}} \tag{113}$$

where the $\frac{P^i K}{Y_H}$ can be empirically determined. Moreover, ϕ can be calculated from (98) and $\frac{K}{\widetilde{K}}$ can be obtained by rearranging (46) to

$$\frac{K}{\widetilde{K}} = \left[(1-\alpha_K)^{1/\eta_K} + \alpha_K^{1/\eta_K}(\frac{K_G}{K})^{\frac{\eta_K-1}{\eta_K}}\right]^{-\eta_K/(\eta_K-1)}$$

where $\frac{K_G}{K}$ is taken from the data. Hence, the equation (113) then only depends on α_K and variables that are already known. As a consequence, we can numerically find a value for α_K such that the equation holds for a given α and η_K. In doing so, we obtain the empirical capital-to-output ratio in the model. In other words, we have used α_K to pin down the capital-to-output ratio in the model for a given, set apriori, value of α. Having set α_K we can determine $\frac{K}{\widetilde{K}}$ from the formula above.

9. **Wage and output:** Using the the expression for marginal cost from (48) we obtain w_t. The employment rate in the private and public sector, N^P and N^G are taken from the data and considered given in the steady-state calibration. Using the first-order condition for optimal labor demand by domestic firms, (96), we obtain Y_H.

10. **Exports:** Having identified the export share in the data $\frac{P_x^* QX}{YCPI}$, we set $X = \frac{P_x^* QX}{YCPI}\frac{YCPI}{Y_H}Y_H/(P_x^* Q)$. Note, that $\frac{Y_H}{YCPI}$ does not directly follow from data but can be constructed from data using P_x^* and Q which are known at this point: $\frac{Y_H}{YCPI} = \frac{P_h Y_H^D}{YCPI}/P_h + \frac{P_x^* QX}{YCPI}/(P_x^* Q)$, where $\frac{P_h Y_H^D}{YCPI} = 1 - \frac{P_x^* QX}{YCPI} - \frac{(1+\tau_t^{SS,F})w^G N_t^G}{YCPI} - \frac{\delta_{KG} K_G P^i}{YCPI} - \frac{P_h INV}{YCPI}$. Using equation (53), we then chose Y^*, such that the imposed level of X is consistent with foreign demand, i.e.

$$Y^* = X/\left((P_x^*)^{-\eta_x}\right).$$

Equation (79) then yields an expression for domestically sold home production, namely

$$Y_H^D = Y_H - X.$$

11. **Government wages and total GDP** To obtain government wages, we obtain the GDP components $\frac{Y_H}{YCPI}$, $\frac{(1+\tau_t^{SS,F})w^G N_t^G}{YCPI}$, $\frac{\delta_{KG} K_G P^i}{YCPI} = \frac{P^i I^G}{YCPI}$ and $\frac{P_h INV}{YCPI}$ empirically. Then it follows, that

$$w^G = \frac{(1+\tau_t^{SS,F})w^G N^G}{YCPI}\left(\frac{Y_H}{YCPI}\right)^{-1}\frac{Y_H}{N^G}\frac{1}{1+\tau_t^{SS,F}}.$$

Using the residual ratio $\frac{P_h INV}{Y}$, we can then determine Y and INV using the definition of Y in (80).

12. **Capital and investment:** Using the first-order condition for private capital demand from (97), one can obtain K. Then, $I = \delta K$ follows from (13). The public capital stock is set, such that the already assumed ratio between public and private capital stock holds, i.e. $K_G = \frac{K_G}{K}K$. The depreciation rate of public capital is set,

such that the implied steady-state public investment to GDP ratio, $\frac{P^i I^G}{YCPI}$, is met, i.e. $\delta_{KG} = \frac{P^i I^G}{YCPI}\frac{1}{P^i}/\frac{K_G}{YCPI}$ as implied by (71) and $A^{I^G} = I^G$ from equation (73). Note, that δ_{KG} and KG can be determined without the knowledge of Y, which is necessary since they are themselves needed for the determination of Y above. Oil sector investment is set such that the corresponding empirical GDP-share holds, i.e. $I^{OIL} = \frac{P^i I^{OIL}}{YCPI}\frac{YCPI}{P^i}$. Total investments, $\widehat{I}$, are then determined by (39), and imported as well as domestically produced investments, I_H and I_F, follow from (41) and (42).

13. **Government spending:** Unemployment benefits are given by $UB = \frac{UB}{YCPI}Y^{CPI}$ where $\frac{UB}{YCPI}$ is taken from the data. Analogously, we set $C^G = \frac{P_h C^G}{YCPI}\frac{YCPI}{P_h}$ where $\frac{P_h C^G}{YCPI}$ is the empirically determined share of government purchases in GDP.

14. **Consumption:** Using the definition of Y_H^D, combined with (36), we obtain

$$C = \frac{Y_H^D - C^G - I_H}{(1-\alpha_c)P_h^{-\eta_C}}.$$

From this, we easily derive C_H and C_F from (36) and (37). Assuming $C = C^o = C^r$ (which we will later show to hold), λ follows from the steady-state version of equation (88), i.e.

$$\lambda = \frac{C^{-\sigma}}{1+\tau^C}.$$

Following the steady-state version of (94), it follows that $\mu = \lambda P_i$.

15. **Employment and participation:** The labor force participation L is empirically determined and assumed to be given. As already discussed, this also applies to N^P and $N = N^P + N^G$.[31] In order to to make the choice of L and N^P model consistent, we choose the parameters θ and ϵ_W such that the labor force participation constraint from (10) holds, as well as steady-state version of the dynamic wage setting equation (20). In doing so, we can also determine the shadow price of being in the work-force, λ^L, from (92). Hence, we set

$$\begin{aligned} \theta &= \lambda(1-(\tau^{OI}+\tau^{BT}+\tau^{SS,H}))(wN^P + w^G N^G) + UB(L-N))/(NL^{\psi}) \\ \lambda^L &= (\lambda UB)/(\lambda UB - N\theta\psi L^{\psi-1}) \end{aligned}$$

The parameter ϵ_W cannot be determined analytically but has to be found by numerical means solving equation (20):

$$\begin{aligned} 0 &= \frac{\theta(N_t)^{\psi}\epsilon_W}{w\lambda} + (1-\tau^W)(1-\epsilon_W) + UB\epsilon_W\frac{1}{w} \\ &+\frac{\lambda^L}{\lambda}\left(\frac{-\epsilon_W}{w}\theta(L^o)^{\psi}\right) - \lambda^L\left((1-\tau^W)(1-\epsilon_W) + UB\frac{\epsilon_W}{w}\right). \end{aligned}$$

Note, that θ is determined independently of the value of ϵ_W, such that there is a unique θ fulfilling the labor force participation constraint. Given this θ, there is a unique ϵ_W fulfilling the wage setting equation.

[31] Note, that due to our assumptions on households aggregation, $L = L^o = L^r$ and $N = N^o = N^r$.

16. **Rule-of-Thumb budget constraint:** As mentioned above, we are assuming that $C = C^r = C^o$ (in the steady state only). To ensure this is the case, we choose lump-sum transfers to rule-of-thumb households, TR^r, in such a way, that $C^r = C$, i.e. from equation (23) it follows

$$TR^r = C(1+\tau^C) - (1-(\tau^O I + \tau^B T + \tau^S SH))(wN^{r,P} + w^G N^{r,G}) - (L^r - N^r)UB.$$

Following the aggregation rules, it then follows $C^o = C^r = C$. Using an empirical aggregate transfer to GDP-ratio, TR/Y^{CPI}, we set $TR = (TR/Y^{CPI})Y^{CPI}$. Using the aggregation equation (33), we can then derive lump-sum transfers to optimizing households. Hence, we chose the aggregate level of transfers according to the data and derive the necessary split between TR^r and TR^o such that consumption of rule-of-thumb and optimizing households are equal.

17. **Other:** The following variables follow directly from their respective equations. Imports IM are given by (57), net exports NX by (82), profits Π from (61) and dividends DIV from (62).

18. **Government budget constraint and balance of payments:** Given empirical targets $\frac{D}{Y^{CPI}}$ and $\frac{QB}{Y^{CPI}}$ we set $D = \frac{D}{Y^{CPI}}Y^{CPI}$ and $B = \frac{QB}{Y^{CPI}}\frac{Y^{CPI}}{Q}$. In order for the balance of payments to hold, we solve (83) for OILR and derive

$$OILR = BQ(1 - R^*\phi(A)/\pi^*) - \frac{Q}{P_f} IM\tau^{IM} - NX - P^i I^{OIL}.$$

The government budget constraint from equation (66) can then be resolved to obtain T^L, since all other components of the budget constraint are known at this point. In other words, we chose the fund withdrawals as an instrument to obtain balance of payments and the lump-sum tax to balance the government budget constraint.

6.8 Derivation of the Market Clearing Condition

In the following we derive the good market clearing, starting from the budget constraint of optimizing households given by equation (8)

$$\begin{aligned}
& C_t^o(1+\tau_t^C) + P_t^i I_t^o + \gamma_t^o + D_t^o - \frac{1}{\pi_t}D_{t-1}^o + Q_t B_t^o - e_t \frac{P_{t-1}^*}{P_t} B_{t-1}^o \\
= \; & LI_t(1-(\tau_t^{OI} + \tau_t^{BT} + \tau_t^{SS,H})) - T_t^{L,o} + UB_t(L_t^o - N_t^o) \\
& + (R_t^k u_t K_t^o + \frac{1}{\pi_t} D_{t-1}^o (R_{t-1} - 1) + e_t \frac{P_{t-1}^*}{P_t} B_{t-1}^o (R_{t-1}^* \phi_t(A_t) - 1) + DIV_t^o)(1-\tau_t^{OI}) \\
& + \tau_t^{OI}(\delta_0 P_t^i K_t^o + \Gamma(u_t) P_t^i K_t^o) + TR_t^o,
\end{aligned}$$

where we have expanded the terms of ordinary income and taxes paid by optimizing households. Multiplying the overall expression by $(1-\omega)$ and inserting the aggregate transfer equation (33), we obtain

$$\begin{aligned}
& C_t^o(1+\tau_t^C)(1-\omega) + P_t^i I_t + \gamma_t + D_t - \frac{1}{\pi_t}D_{t-1} + Q_t B_t - e_t \frac{P_{t-1}^*}{P_t} B_{t-1} \\
= \; & (1-\omega) LI_t(1-(\tau_t^{OI} + \tau_t^{BT} + \tau_t^{SS,H})) - T_t^L + (1-\omega) UB_t(L_t^o - N_t^o) \\
& + (R_t^k u_t K_t + \frac{1}{\pi_t} D_{t-1} (R_{t-1} - 1) + e_t \frac{P_{t-1}^*}{P_t} B_{t-1} (R_{t-1}^* \phi_t(A_t) - 1) + DIV_t)(1-\tau_t^{OI}) \\
& + \tau_t^{OI}(\delta_0 P_t^i K_t + \Gamma(u_t) P_t^i K_t) + TR_t - \omega TR_t^r.
\end{aligned}$$

Now, we insert the Rule-of-thumb household's budget constraint which yields

$$\begin{aligned}
& C_t^o(1+\tau_t^C)(1-\omega) + P_t^i I_t + \gamma_t + D_t - \frac{1}{\pi_t} D_{t-1} + Q_t B_t - e_t \frac{P_{t-1}^*}{P_t} B_{t-1} \\
= \;& (1-\omega) LI_t (1 - (\tau_t^{OI} + \tau_t^{BT} + \tau_t^{SS,H})) - T_t^L \\
& + \omega((1-\tau_t^W)(w_t N_t^{r,P} + w_t^G N_t^{r,G}) + UB_t(L_t^r - N_t^r) - C_t^r(1+\tau_t^C)) + (1-\omega) UB_t (L_t^o - N_t^o) \\
& + (R_t^k u_t K_t + \frac{1}{\pi_t} D_{t-1}(R_{t-1} - 1) + e_t \frac{P_{t-1}^*}{P_t} B_{t-1}(R_{t-1}^* \phi_t(A_t) - 1) + DIV_t)(1-\tau_t^{OI}) \\
& + \tau_t^{OI}(\delta_0 P_t^i K_t + \Gamma(u_t) P_t^i K_t) + TR_t.
\end{aligned}$$

Using the aggregation rules from section 2.2.3, we obtain

$$\begin{aligned}
& C_t(1+\tau_t^C) + P_t^i I_t + \gamma_t + D_t - \frac{1}{\pi_t} D_{t-1} + Q_t B_t - e_t \frac{P_{t-1}^*}{P_t} B_{t-1} \\
= \;& (w_t N_t^P + w_t^G N_t^G)(1 - (\tau_t^{OI} + \tau_t^{BT} + \tau_t^{SS,H})) - T_t^L + UB_t(L_t - N_t) \\
& + (R_t^k u_t K_t + \frac{1}{\pi_t} D_{t-1}(R_{t-1} - 1) + e_t \frac{P_{t-1}^*}{P_t} B_{t-1}(R_{t-1}^* \phi_t(A_t) - 1) + DIV_t)(1-\tau_t^{OI}) \\
& + \tau_t^{OI}(\delta_0 P_t^i K_t + \Gamma(u_t) P_t^i K_t) + TR_t.
\end{aligned}$$

In the next step we replace T_t^L with an expression derived from the government budget constraint from (66), which yields (after several tax terms cancel out)

$$\begin{aligned}
& C_t + P_t^i I_t + \gamma_t + Q_t B_t - e_t \frac{P_{t-1}^*}{P_t} B_{t-1} R_{t-1}^* \phi_t(A_t) \\
= \;& (w_t N_t^P + w_t^G N_t^G) + (w_t N_t^P + w_t^G N_t^G)\tau_t^{SS,F} + (\Pi_{H,t} + \Pi_{X,t})\, \tau_t^{\Pi,F} + OILR_t \\
& - P_{h,t} C_t^G - P_t^i I_t^G - w_t^G N_t^G (1 + \tau_t^{SS,F}) + R_t^k u_t K_t + DIV_t.
\end{aligned}$$

Rearranging and employing the definition of dividends, equation (62), yields

$$\begin{aligned}
& C_t + P_t^i I_t + P_{h,t} C_t^G + P_t^i I_t^G + \gamma_t + Q_t B_t - e_t \frac{P_{t-1}^*}{P_t} B_{t-1} R_{t-1}^* \phi_t(A_t) \\
= \;& w_t N_t^P + w_t N_t^P \tau_t^{SS,F} + (\Pi_{H,t} + \Pi_{X,t}) + OILR_t + R_t^k u_t K_t.
\end{aligned}$$

In the next step we insert the definition of profits from equation (61) and obtain after rearranging

$$\begin{aligned}
& C_t + P_t^i I_t + P_{h,t} C_t^G + P_t^i I_t^G + \gamma_t + Q_t B_t - e_t \frac{P_{t-1}^*}{P_t} B_{t-1} R_{t-1}^* \phi_t(A_t) \\
= \;& P_{h,t} Y_{H,t}^D + Q_t P_{x,t}^* X_t - AC_{H,t} - AC_{X,t} + OILR_t.
\end{aligned}$$

In the following, we apply the balance of payments from equation (83), which yields

$$C_t + P_t^i I_t + P_{h,t} C_t^G + P_t^i I_t^G + NX_t + P_t^i I_t^{OIL} = P_{h,t} Y_{H,t}^D + Q_t P_{x,t}^* X_t - AC_{H,t} - AC_{X,t} - \gamma_t,$$

which, using the definition of GDP (in CPI units), yields our final market clearing condition

$$\begin{aligned}
Y_t^{CPI} &= P_{h,t} Y_{H,t}^D + Q_t P_{x,t}^* X_t + (1+\tau_t^{SS,F})\overline{w_t^G} N_t^G + \overline{P_t^i} \delta_{KG} K_{G,t} + P_{h,t} INV_t \\
&= C_t + P_{h,t} C_t^G + P_t^i (I_t + I_t^G + I_t^{OIL}) + NX_t + (1+\tau_t^{SS,F})\overline{w_t^G} N_t^G + \\
& \quad \overline{P_t^i} \delta_{KG} K_{G,t} + P_{h,t} INV_t + AC_{H,t} + AC_{X,t} + \gamma_t.
\end{aligned}$$

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