

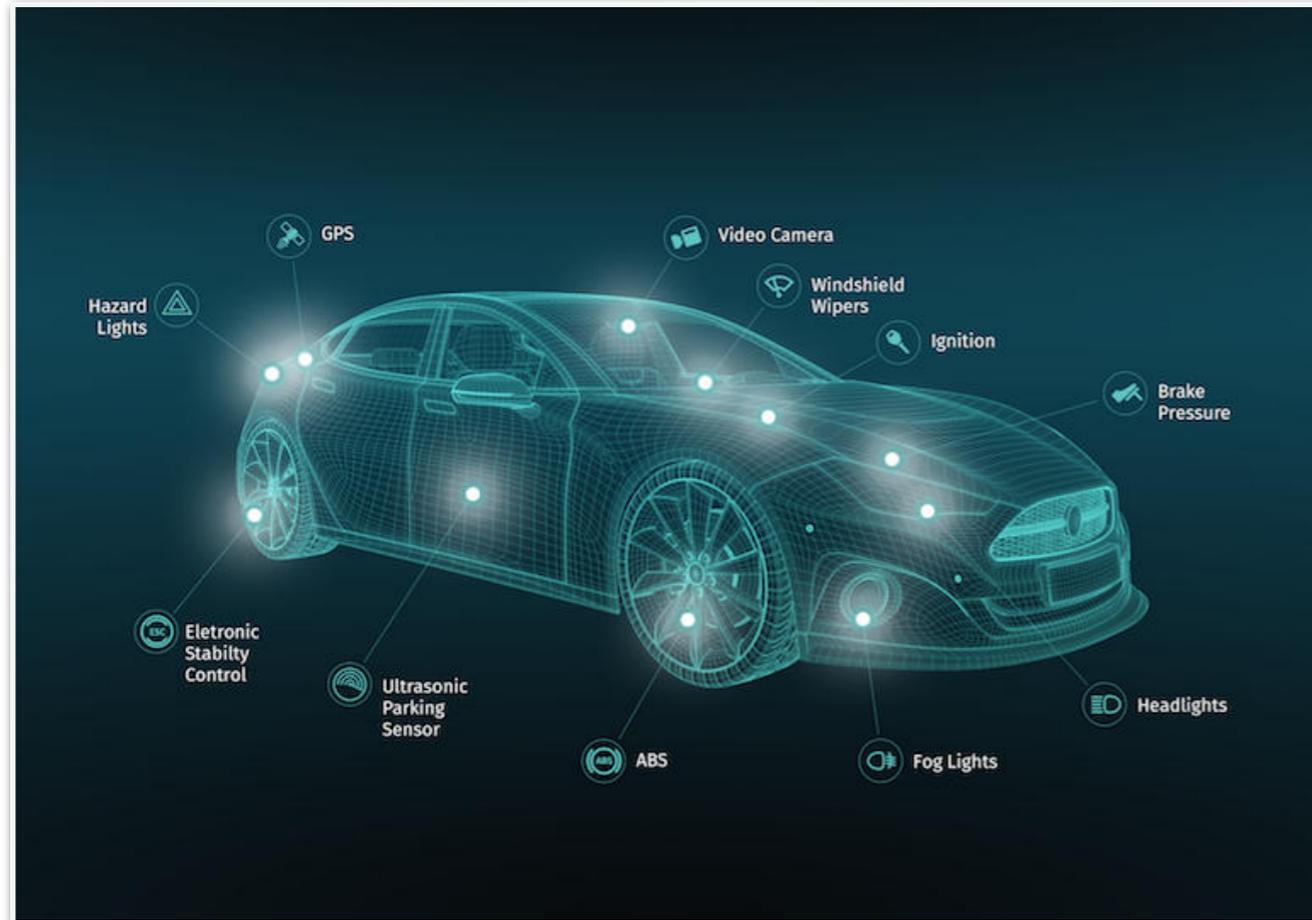
# Learning efficient data representation

**Valeriya Naumova**  
**Machine Intelligence Department**  
**SimulaMet**

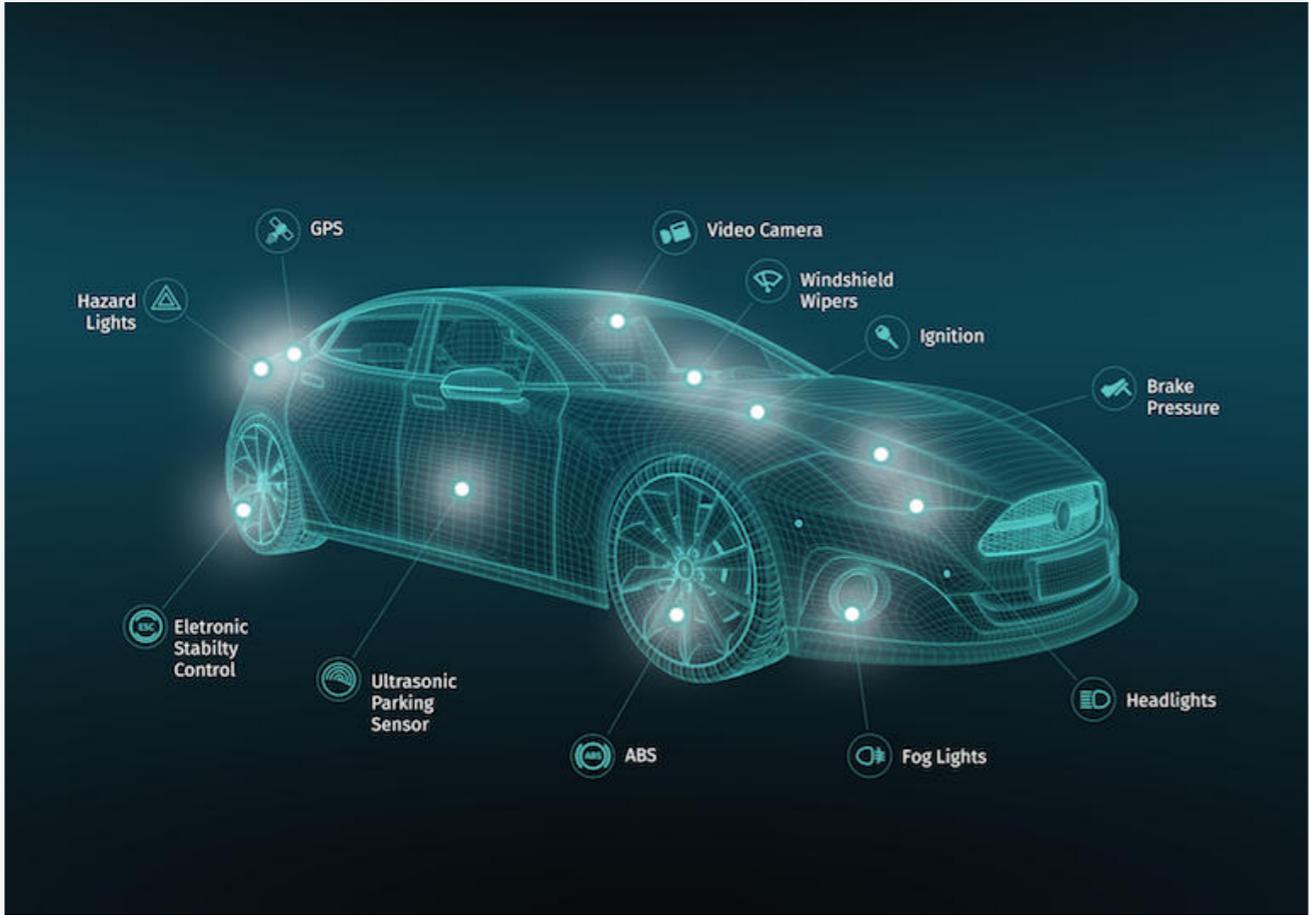
**NORA Kick-Off**  
**April 01, 2019**

**We collect a large amount of (*indirect*) measurements and would like to have efficient tools to analyse and process them**

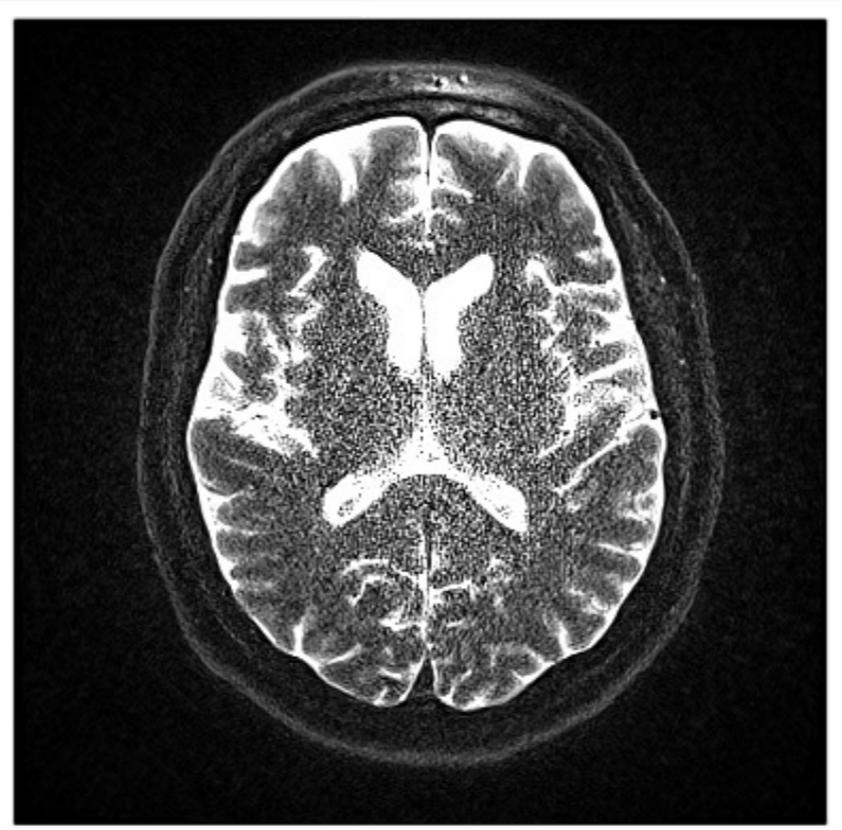
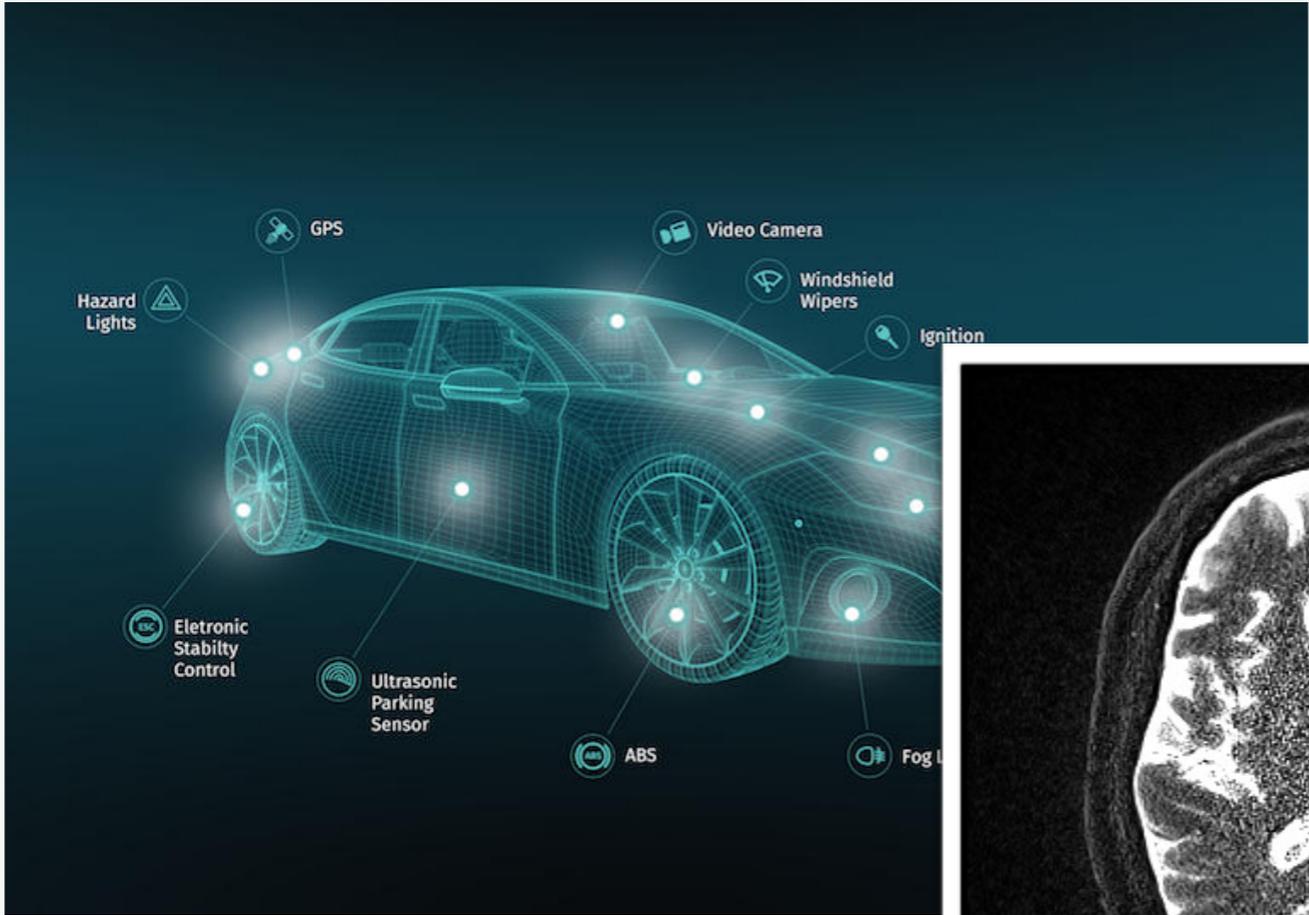
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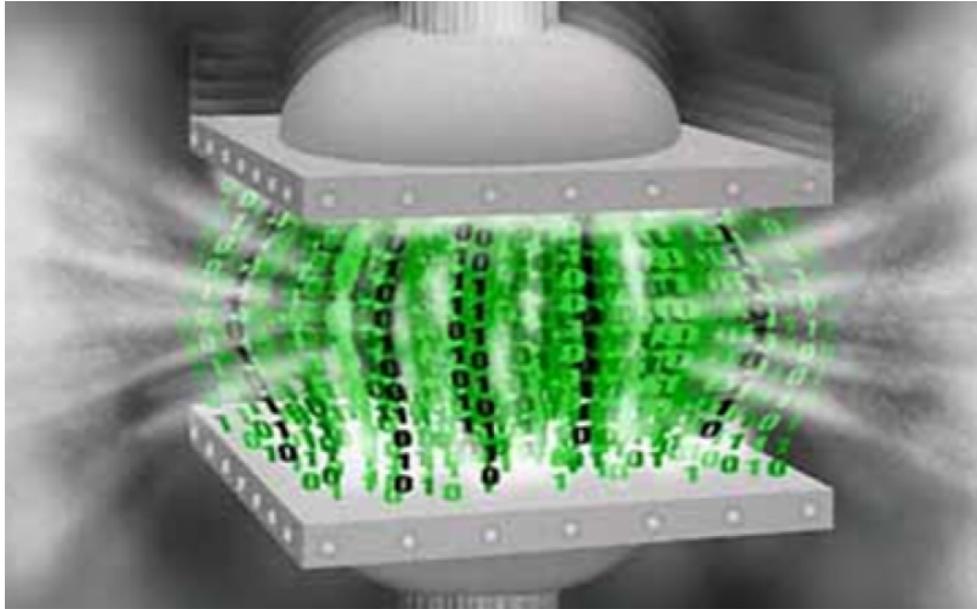
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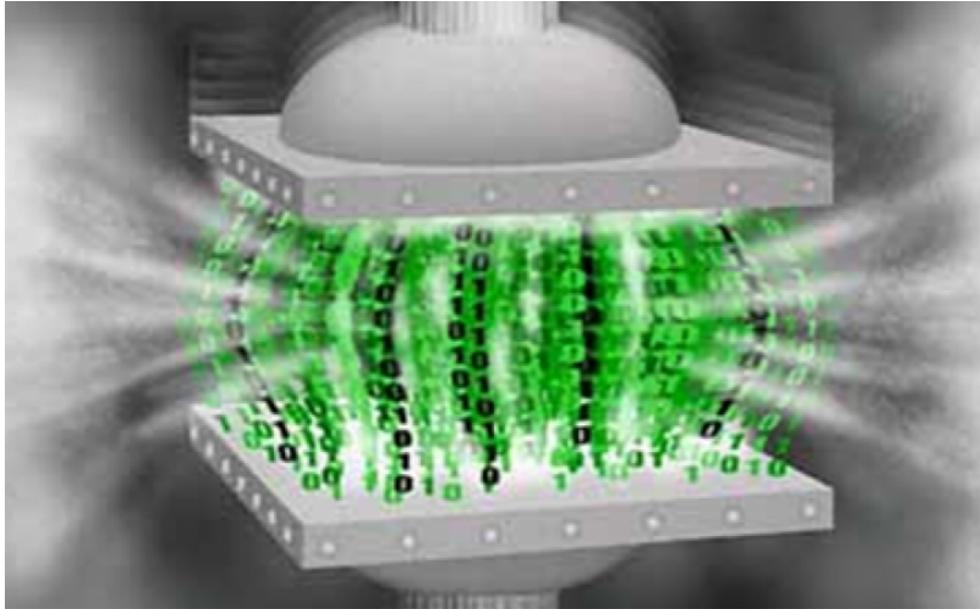


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***Compression***

We collect a large amount of (*indirect*) measurements and would like to have efficient tools to analyse and process them

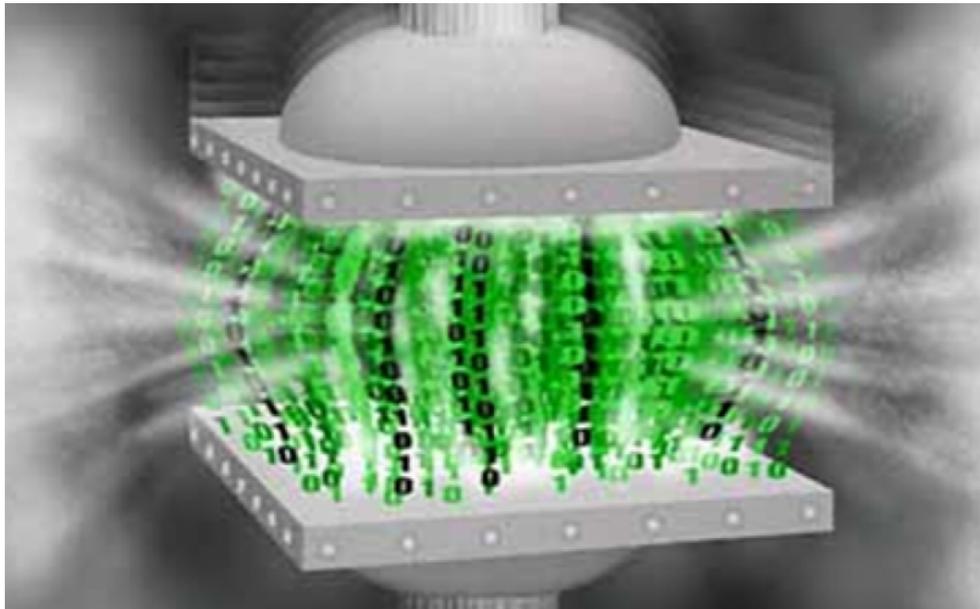


***Compression***



***Segmentation***

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***Compression***

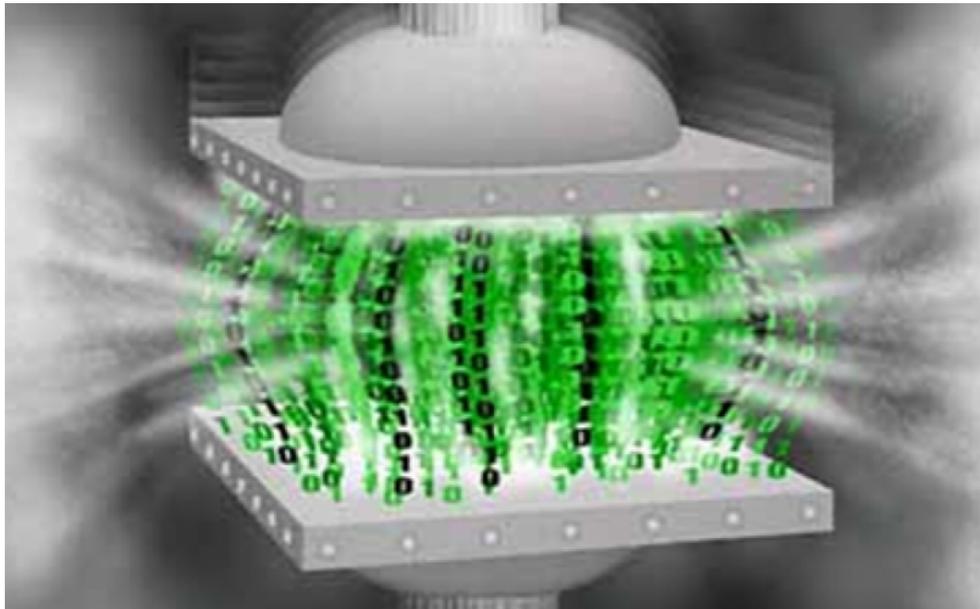


***Segmentation***



***Prediction***

We collect a large amount of (*indirect*) measurements and would like to have efficient tools to analyse and process them



***Compression***



***Segmentation***



***Prediction***



***Classification***

**Recent advances in modern signal processing are based on the fact that high-dimensional data follows a low complexity model**

## **What is sparsity?**

**Sparsity implies many zeros in a vector or a matrix**

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## What is sparsity?

Sparsity implies many zeros in a vector or a matrix



Fingerprint patch

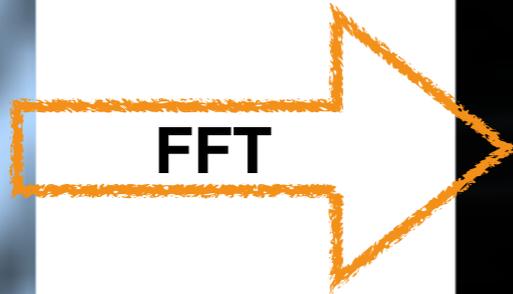
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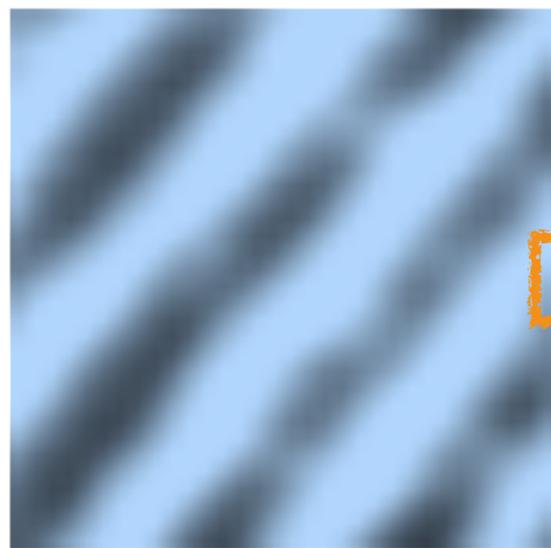


FFT response

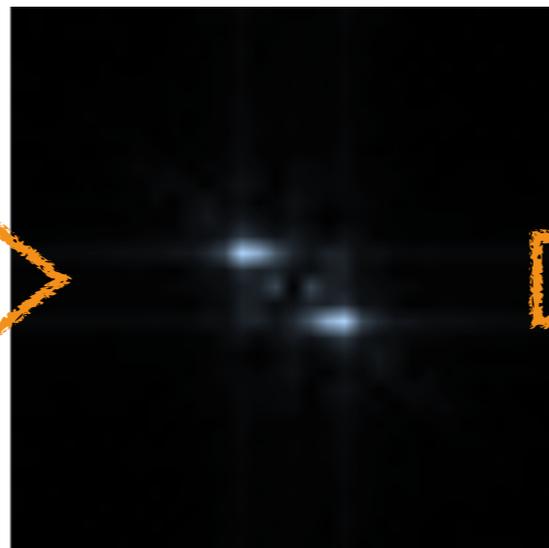
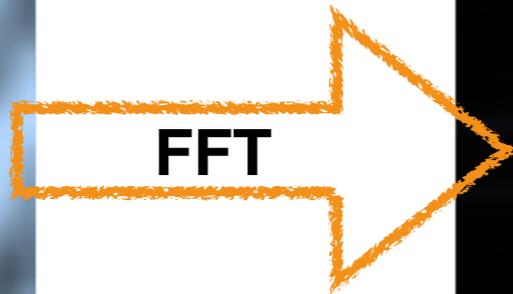
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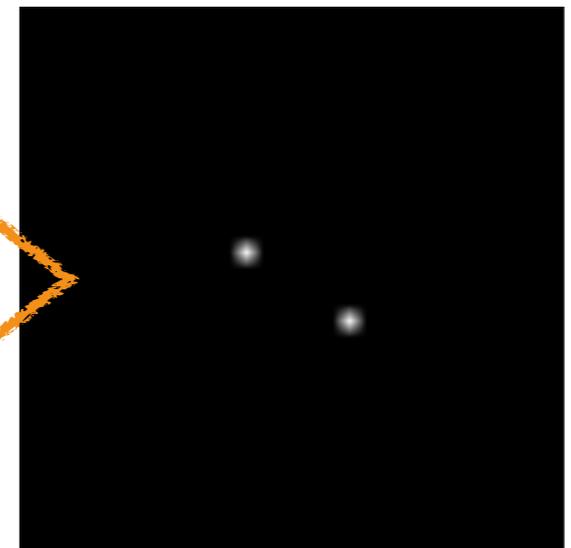
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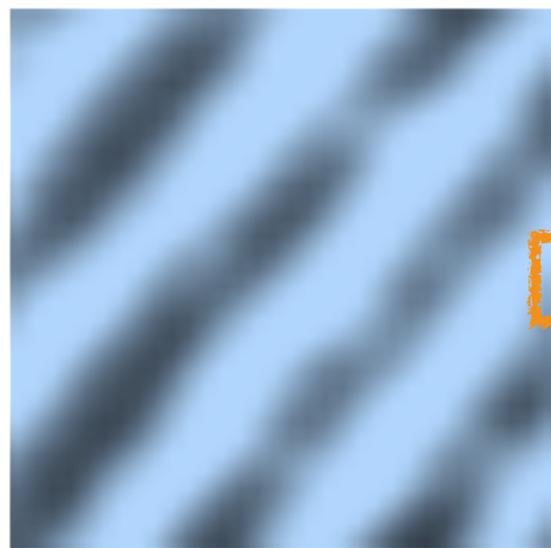


Sparse representation

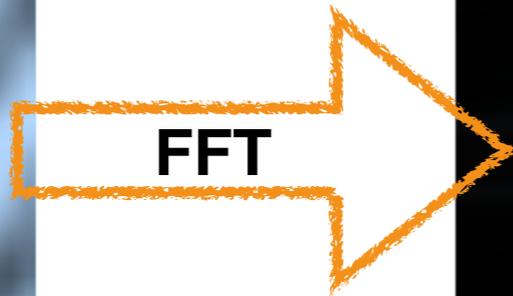
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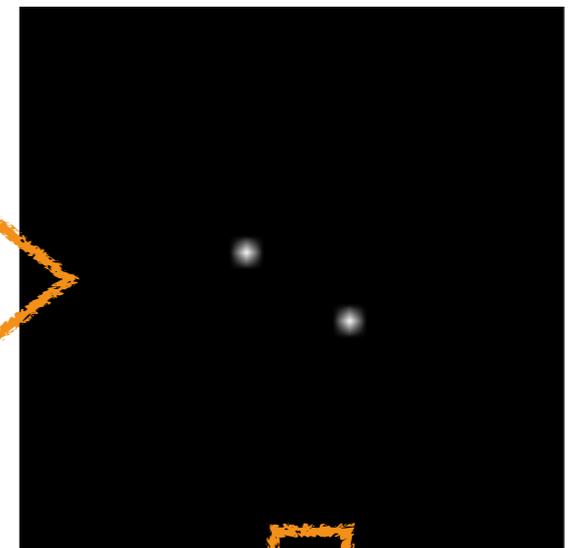
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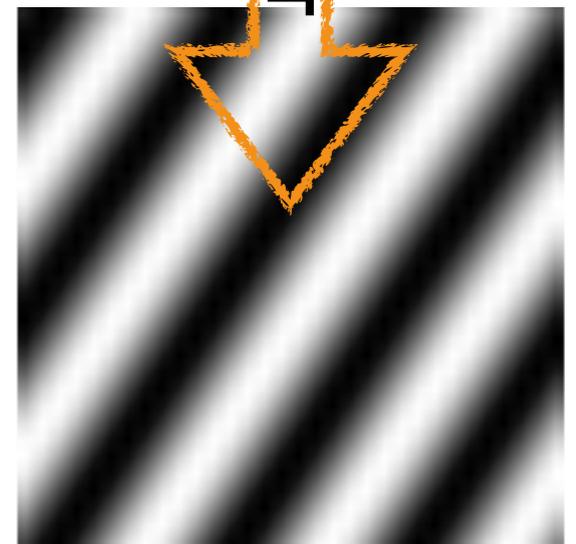
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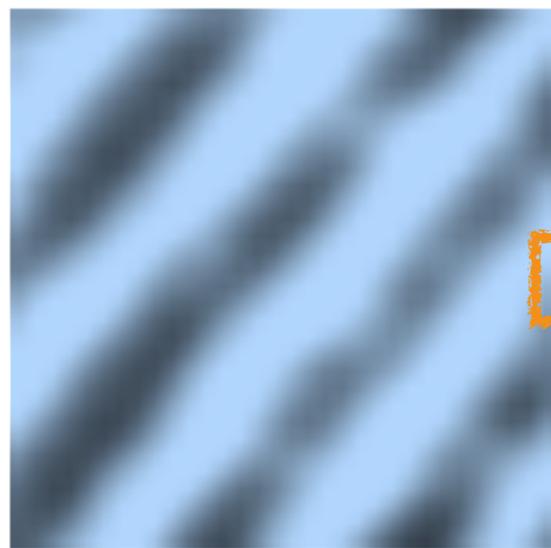


Reconstructed patch

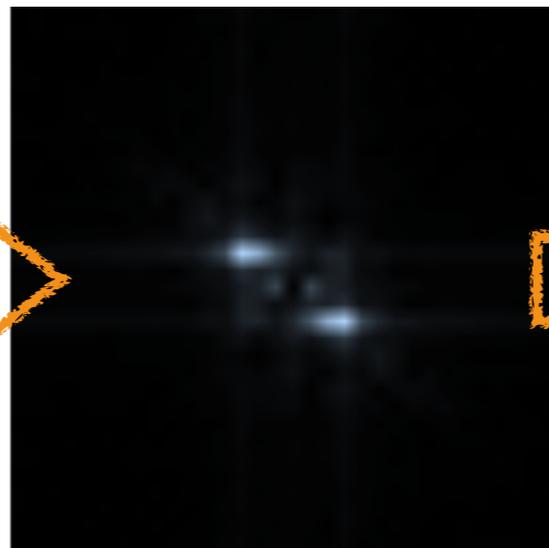
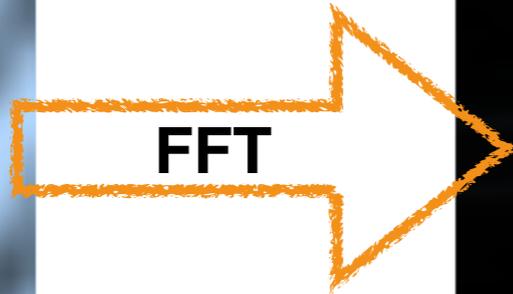
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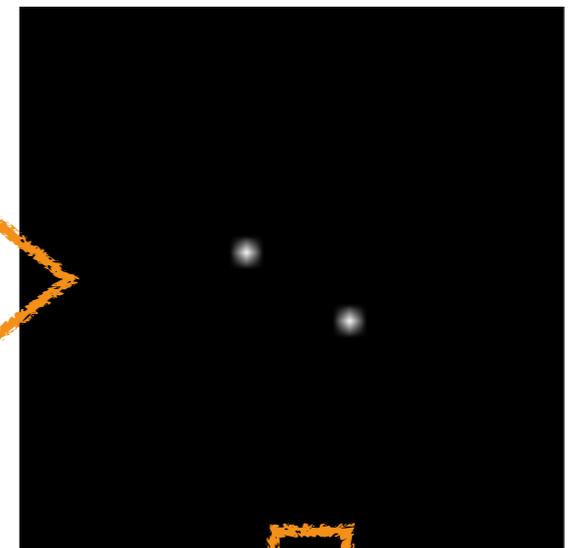
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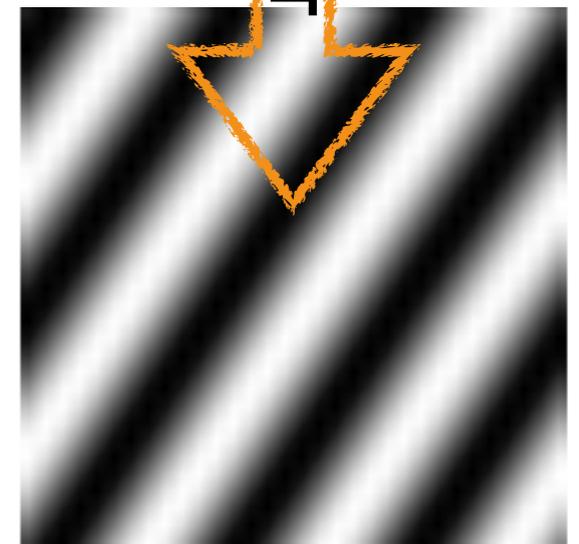
Fingerprint patch



FFT response



Sparse representation



Reconstructed patch

Main idea:

Sparse representation captures the essential information in a signal

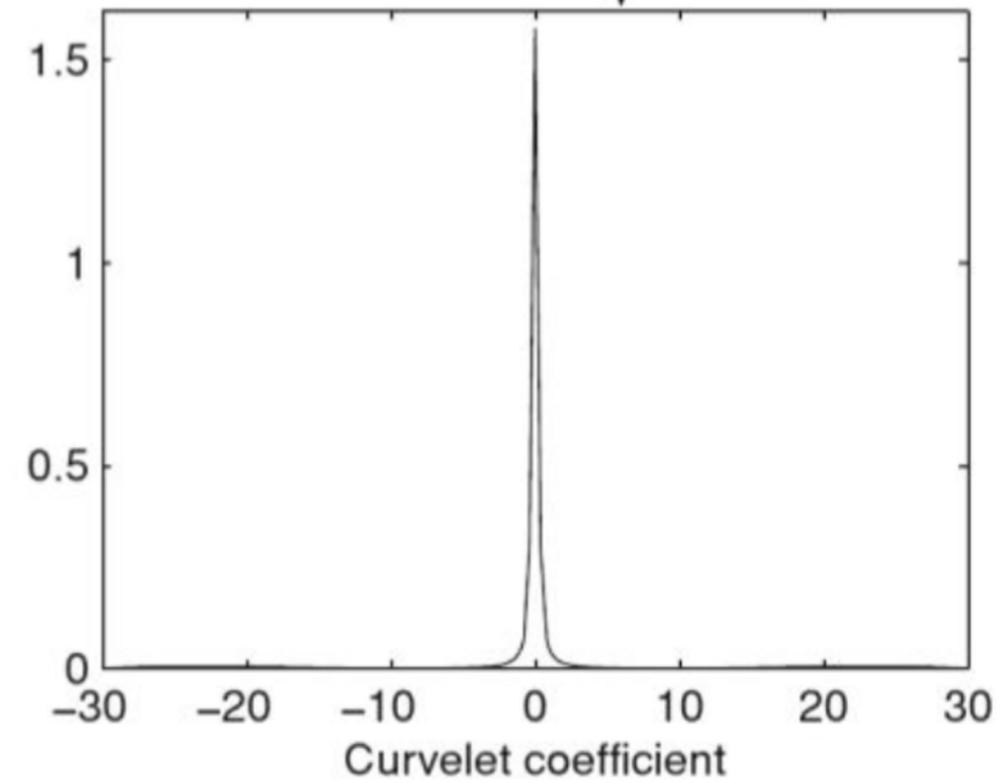
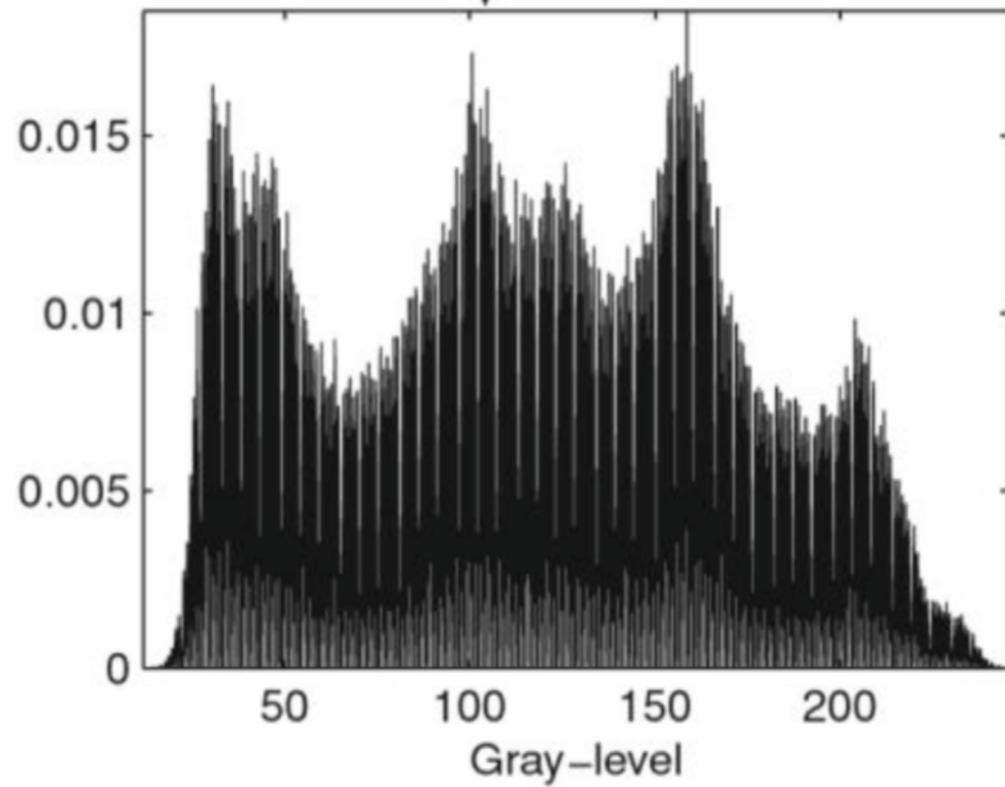
**Are images sparse?**

# Are images sparse?

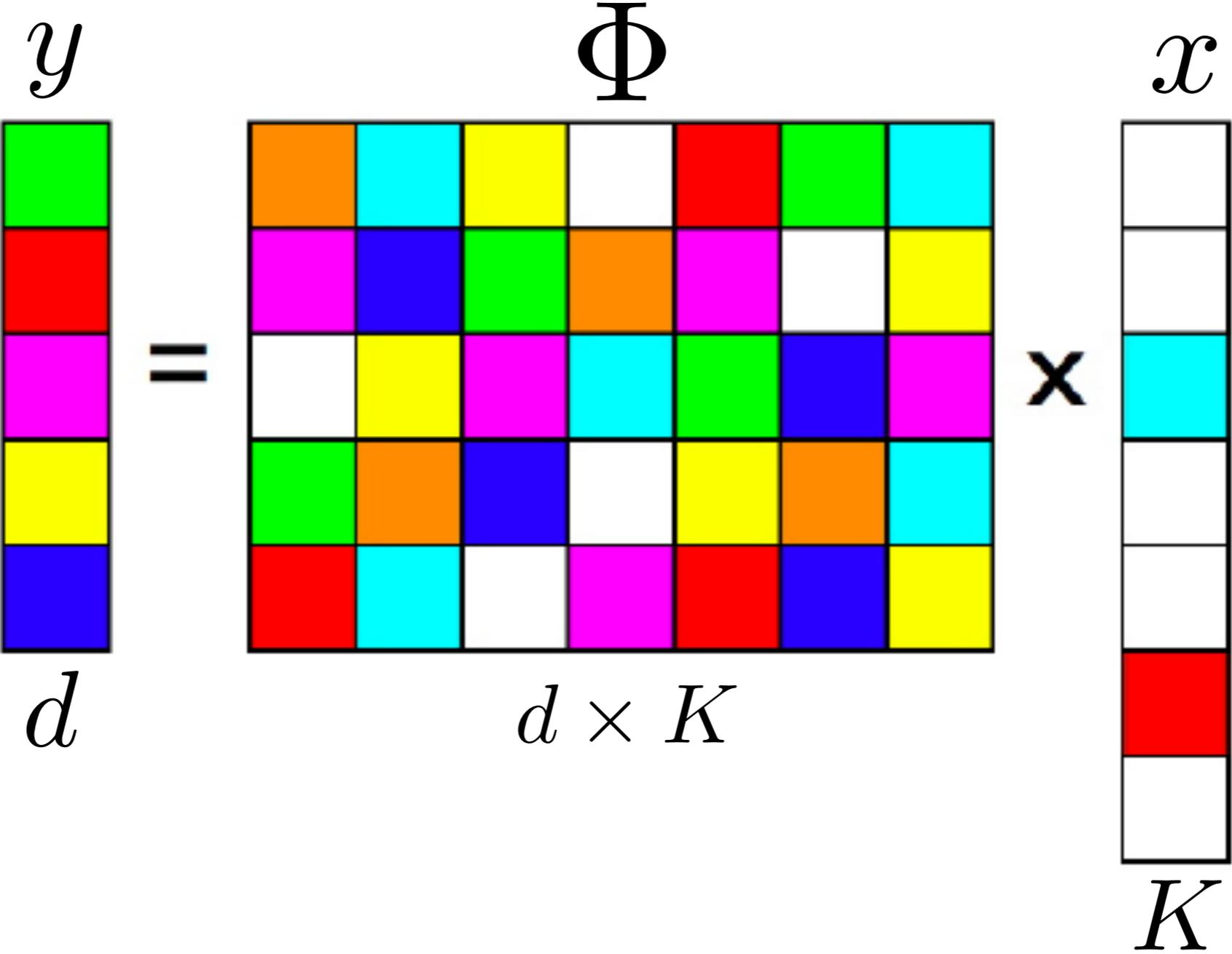


Original domain

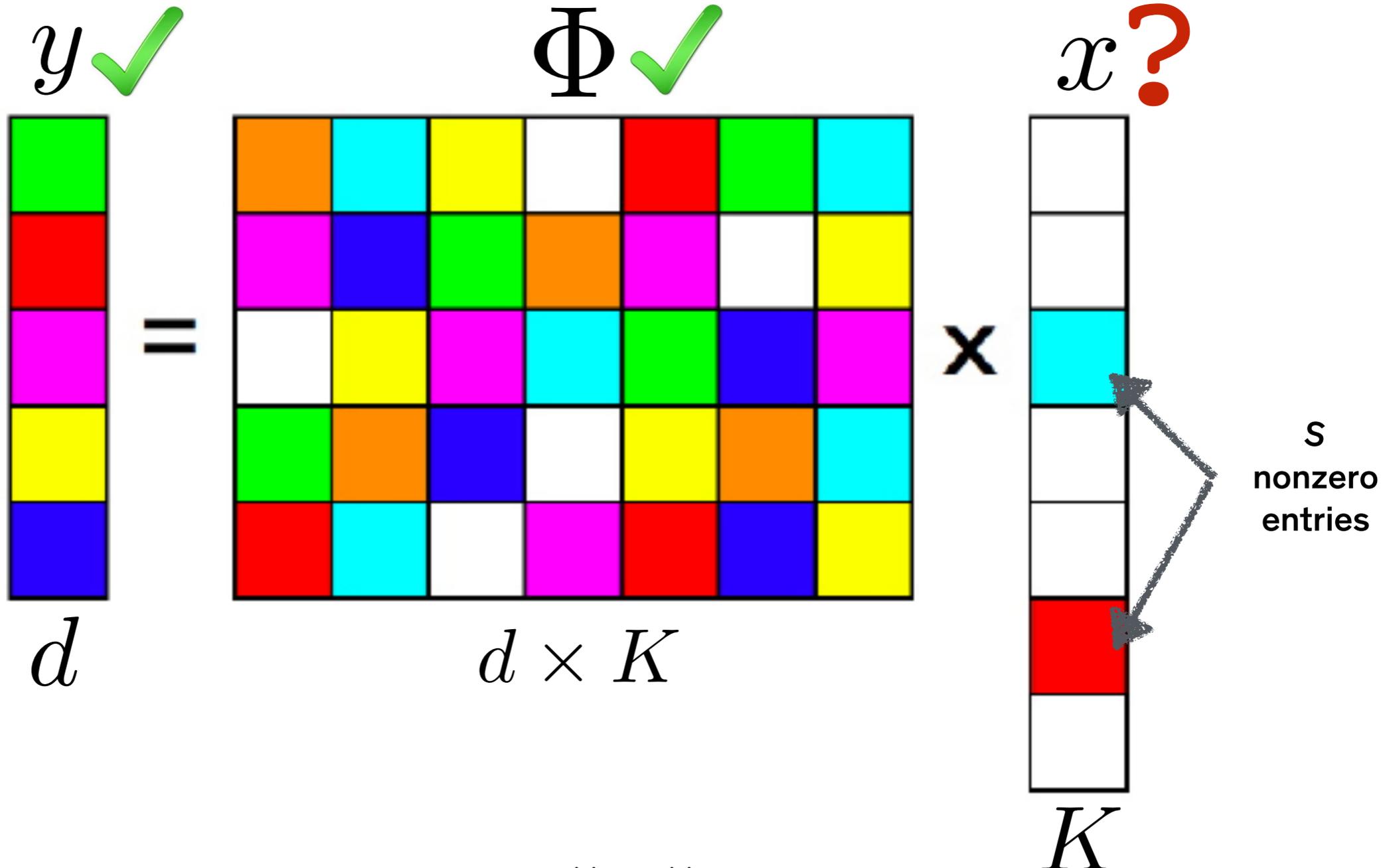
Curvelet domain



Natural signals are often high-dimensional that can be well represented by a small (sparse) number of elementary signals



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$$y = \Phi x, \quad \|x\|_0 \ll d$$



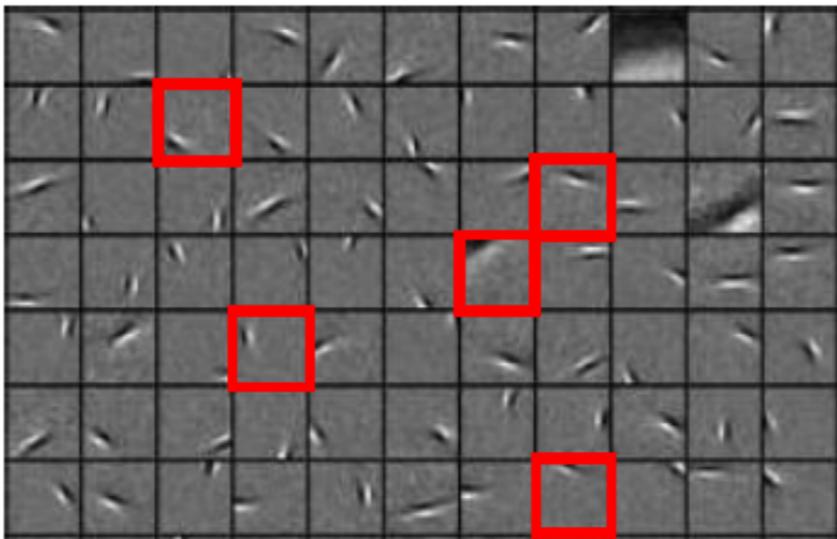
A 'well-learned' dictionary allows to represent every signal of a class using only a small (sparse) number of atoms

$y$  ✓

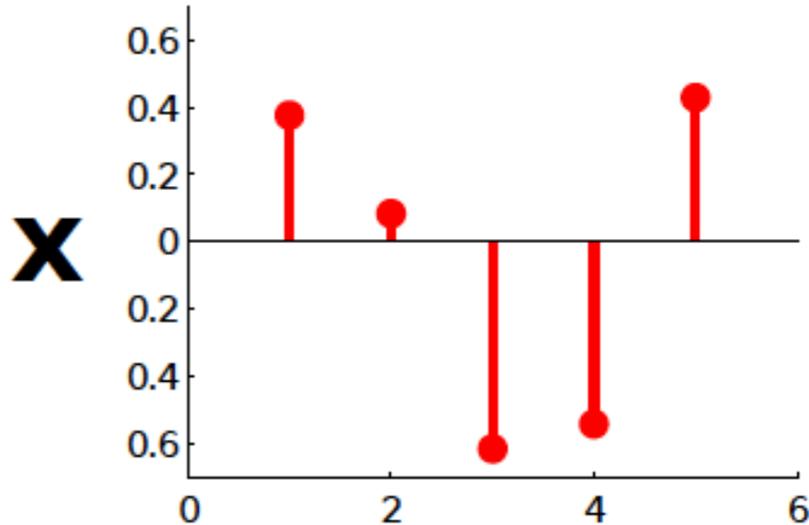


=

$\Phi$  ✓



$x$  ?



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**Designed dictionaries:** Wavelets, Curvelets, Overcomplete Discrete Cosine Transform, ....

[Haar, 1910], [Zweig, Morlet, Grossman '70s], [Meyer, Mallat, Daubechies, Coifman, Donoho, Candes 80s-today], ...

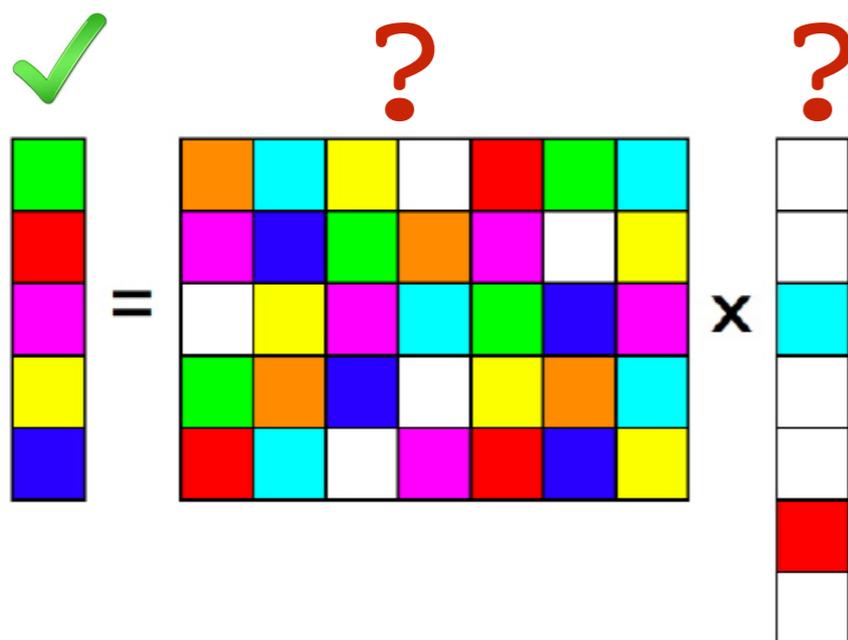
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**Data-driven dictionary learning:**

[Olshausen and Field, 1997], [Engan et al., 1999], [Aharon et al., 2006], [Roth and Black, 2005], [Lee et al., 2007], [Gribonval and Schnass, 2010], [Starck et al., 2013], [Schnass, 2015],....



$$\min_{x_n, \Phi \in \mathcal{C}} \sum_n \underbrace{\frac{1}{2} \|y_n - \Phi x_n\|^2}_{\text{reconstruction}} + \underbrace{\alpha \psi(x_n)}_{\text{sparsity}}$$

- $\psi(x) = \|x\|_0$
- $\psi(x) = \|x\|_1$

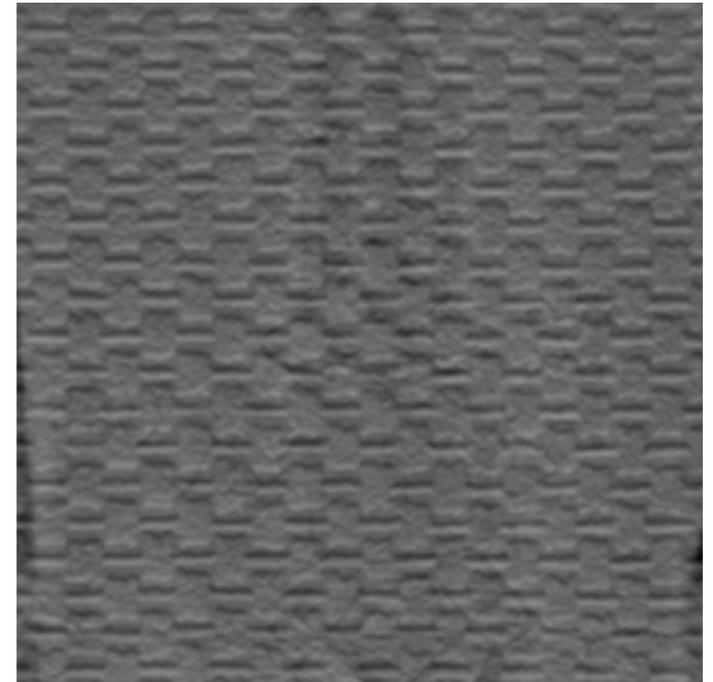
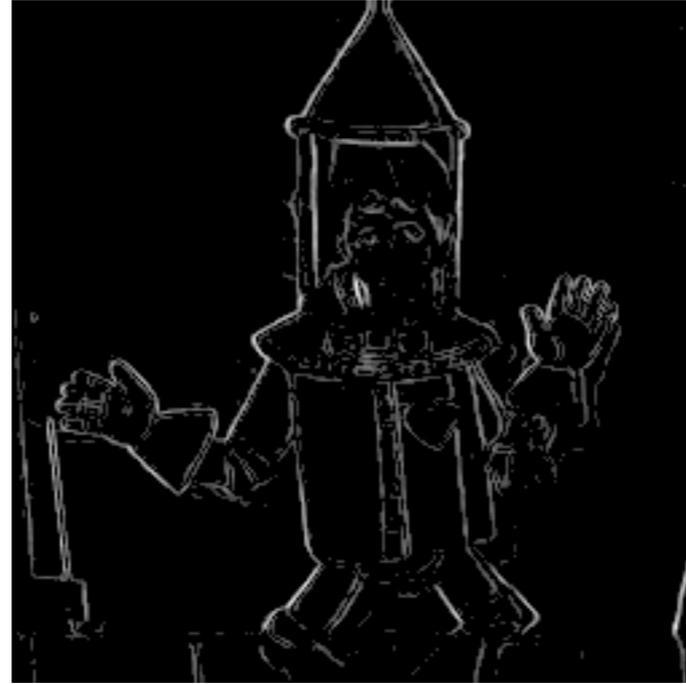
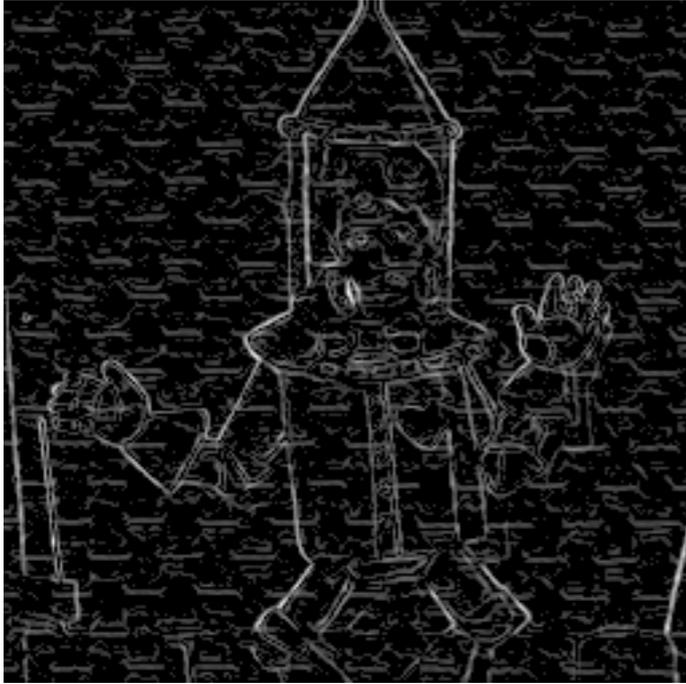
# Dictionary learning has been successfully used in a number of applications like compression



RMSE is shown in brackets

[O. Bryt, M. Elad, 2008]

**Dictionary learning has been successfully used in a number of applications like edge detection and texture separation**

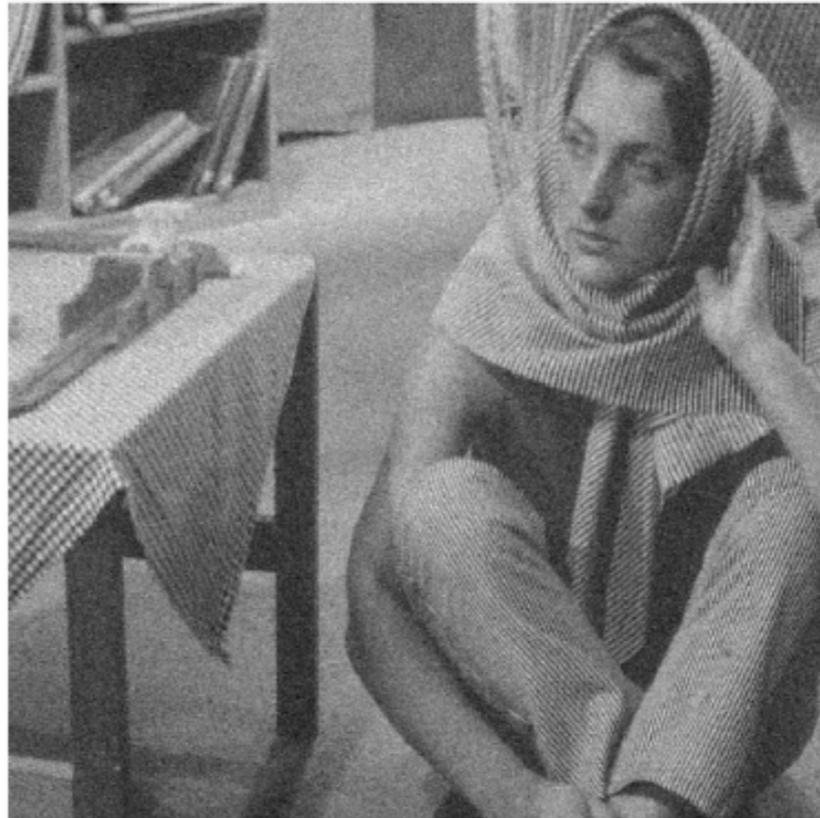


# Dictionary learning has been successfully used in a number of applications like denoising

Original Image



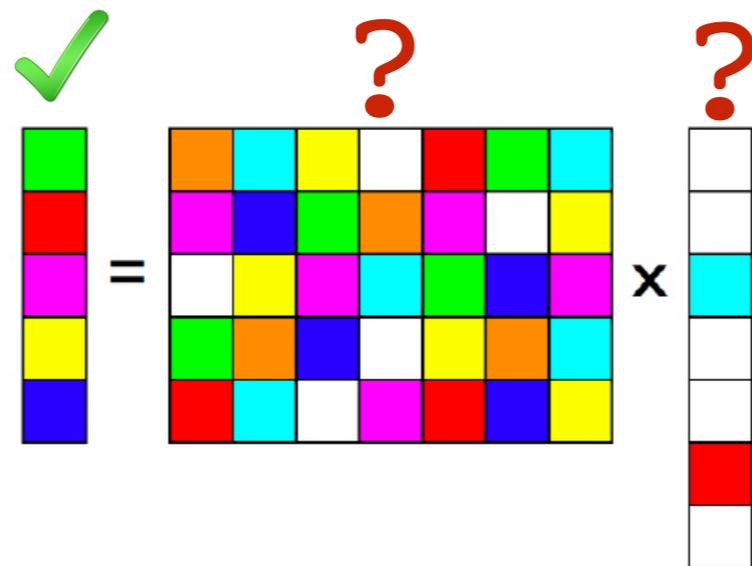
Noisy Image (22.1307 dB,  $\sigma=20$ )



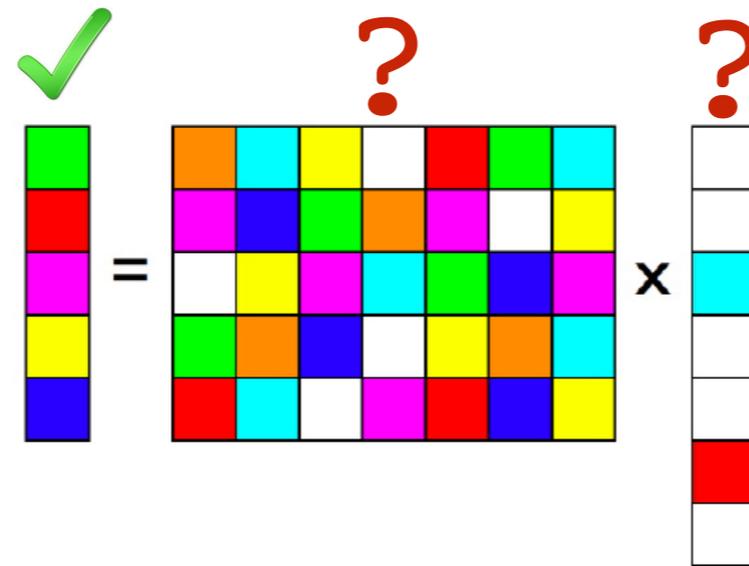
Denoised image (30.83 dB)



Dictionary learning delivers good results BUT only when a large number of clean high-quality signals is available



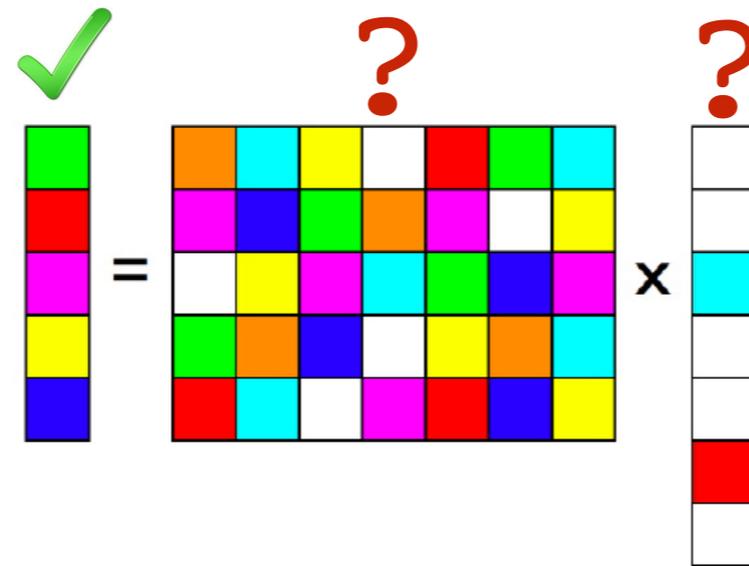
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## Dictionary learning:

- ✓ delivers state-of-the-art results for many image/video processing tasks.
- ✓ is well adapted to data that admits sparse representation.
- **requires a large amount of high-quality** clean signals for training.
- are **computationally demanding**.

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Limited applicability for high-dimensional data.  
Limited applicability for real-life sensor data.

**Can a dictionary be efficiently learned when there are only a few, or no clean, training signals available?**

# Can a dictionary be efficiently learned when there are only a few, or no clean, training signals available?

- ▶ We propose a novel algorithm, *Iterative Thresholding and K-residual Means for Masked data (ITKrMM)*, to solve the problem of learning from incomplete or corrupted data.

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- ▶ ITKrMM algorithm demonstrates *significant improvement in terms of computational complexity* compared to the state-of-the-art methods.

# ITKrMM algorithm has the same reconstruction quality as the state-of-the-art method

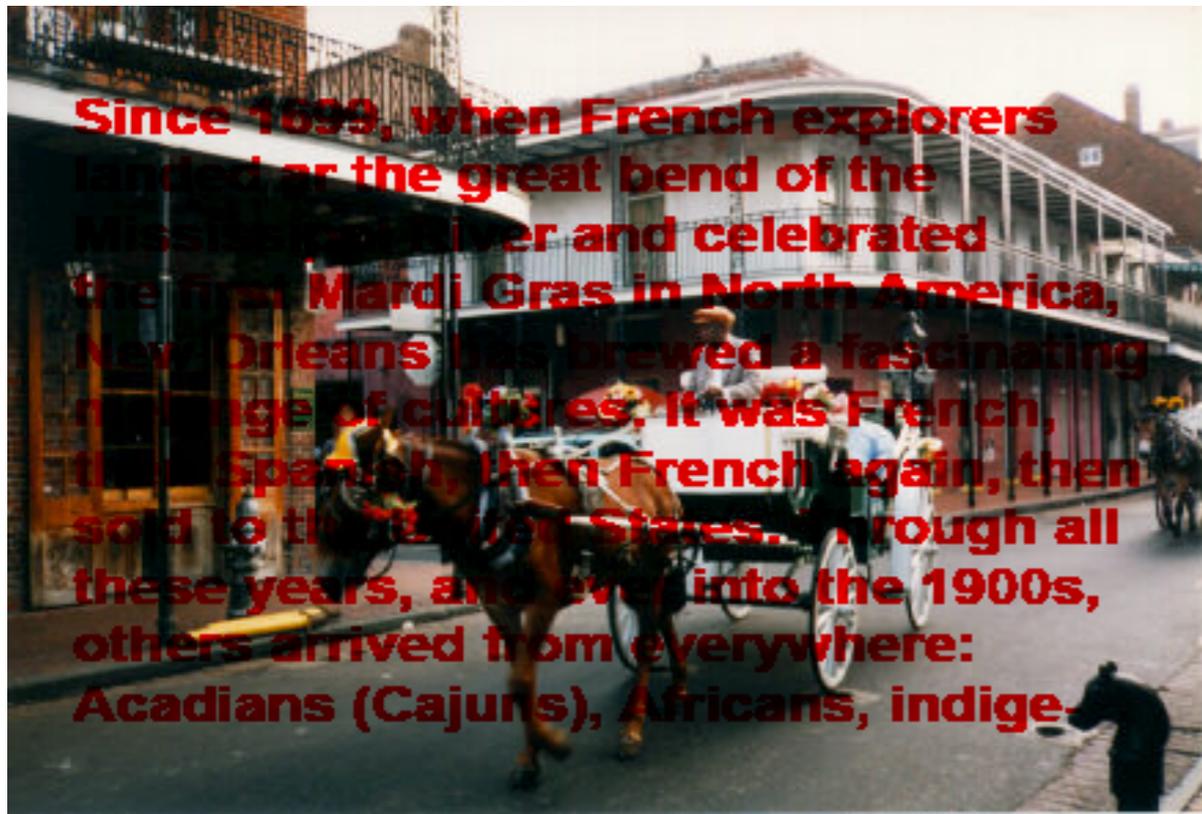


Image corrupted with the text



Recovered image with ITKrMM dictionary

# ITKrMM algorithm has the same reconstruction quality as the state-of-the-art method



Corrupted image with  
70 % missing data

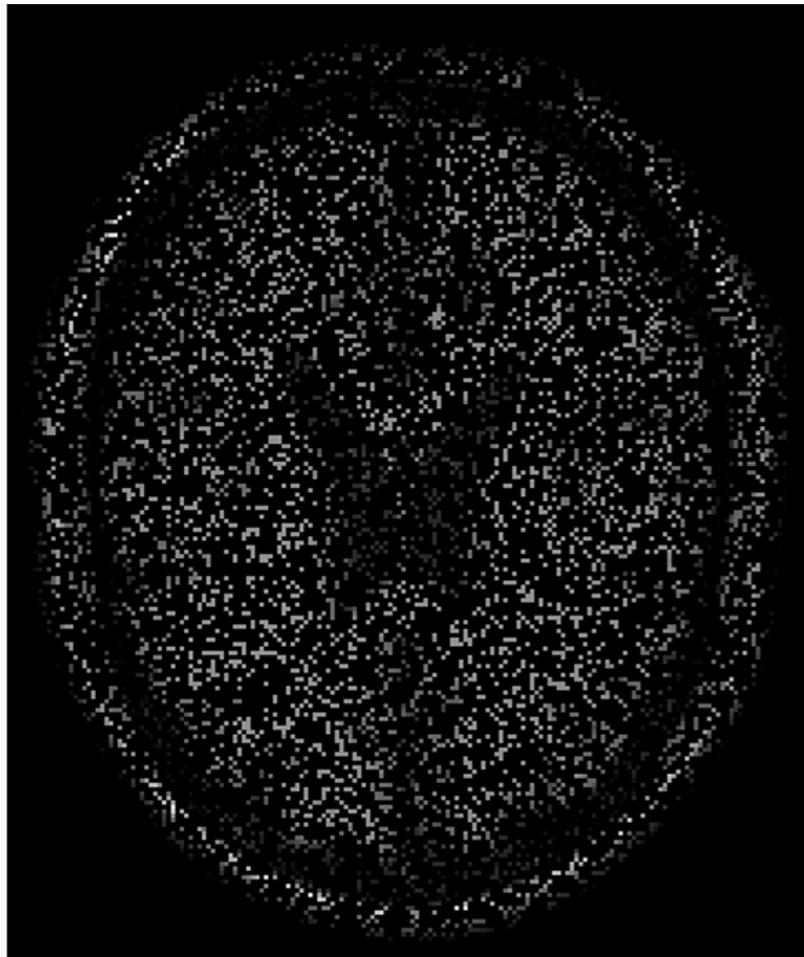


Ground truth

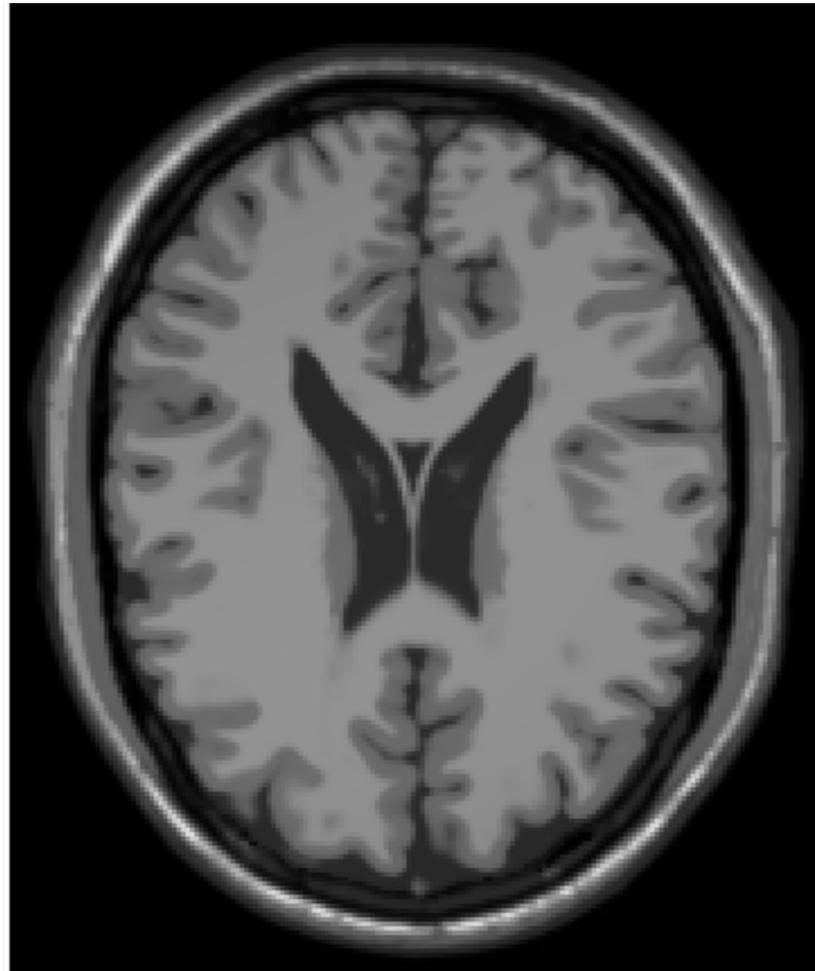


Recovered image with ITKrMM  
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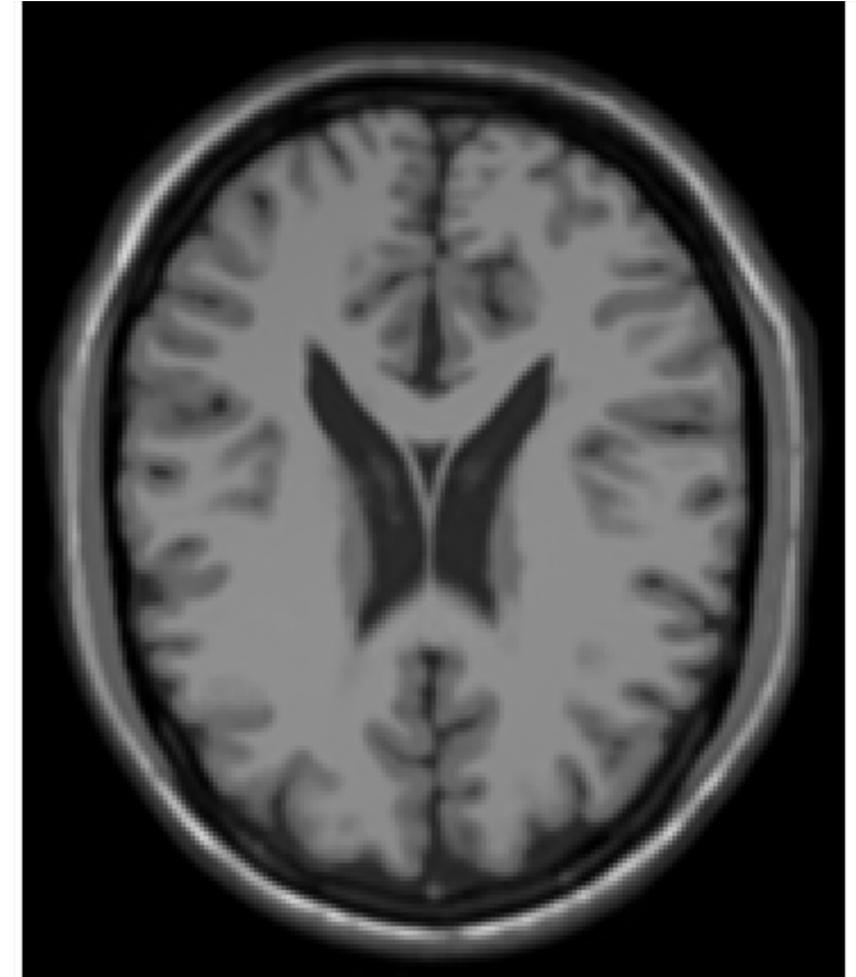
# Good performance and reasonable complexity of ITKrMM also valid for 3D image inpainting



MRI volume with 80 %  
missing voxels

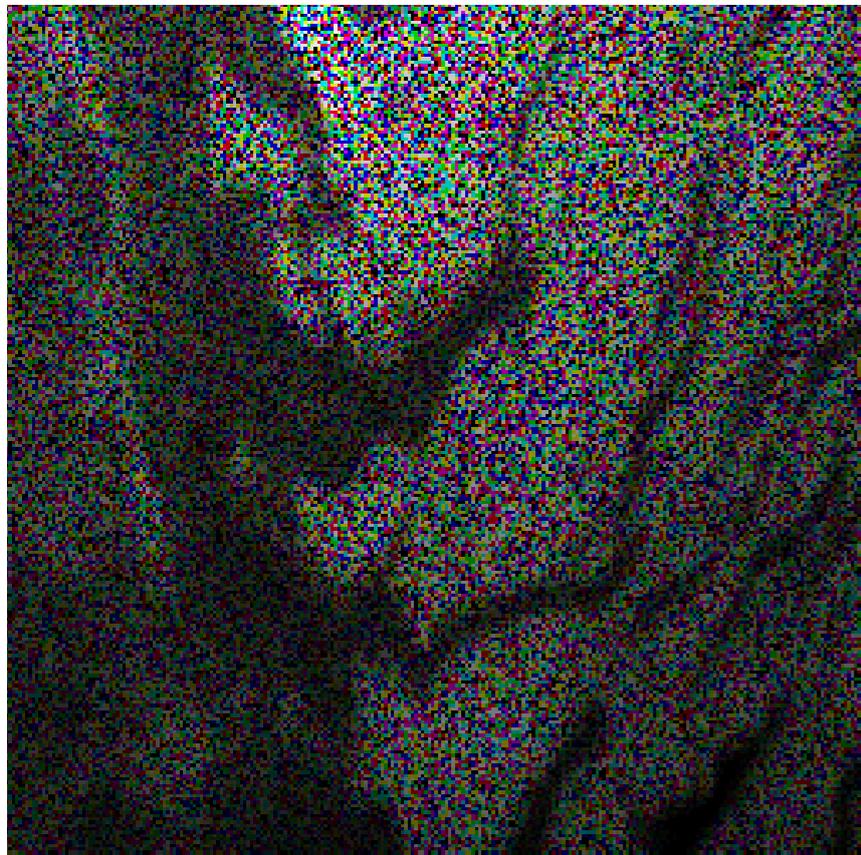


MRI volume of size  
 $217 \times 181 \times 181$

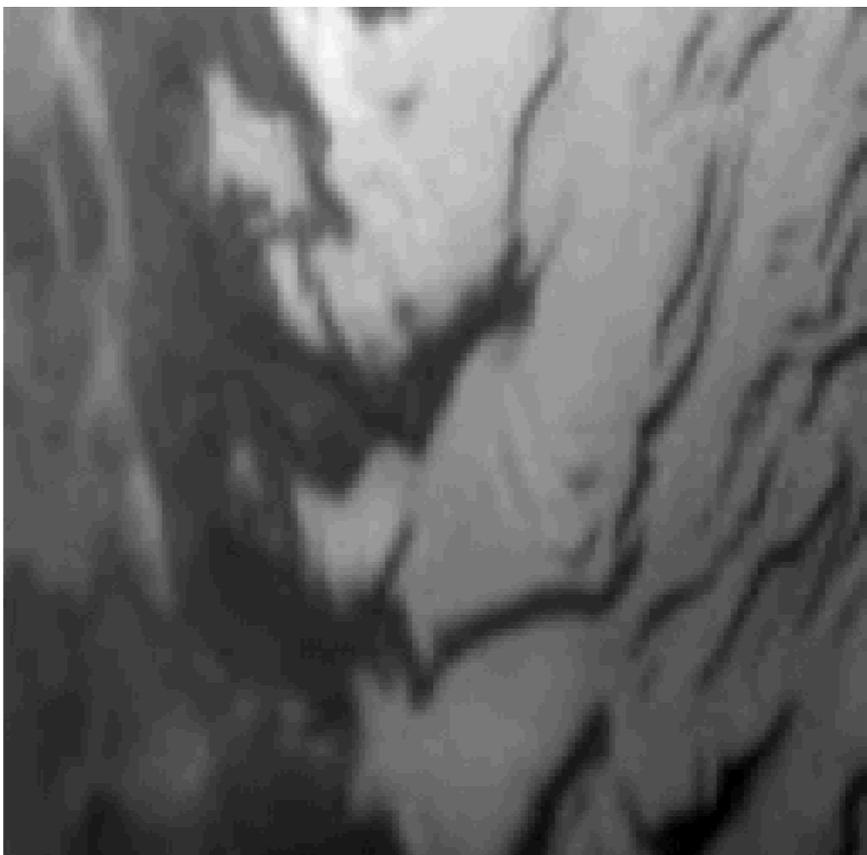


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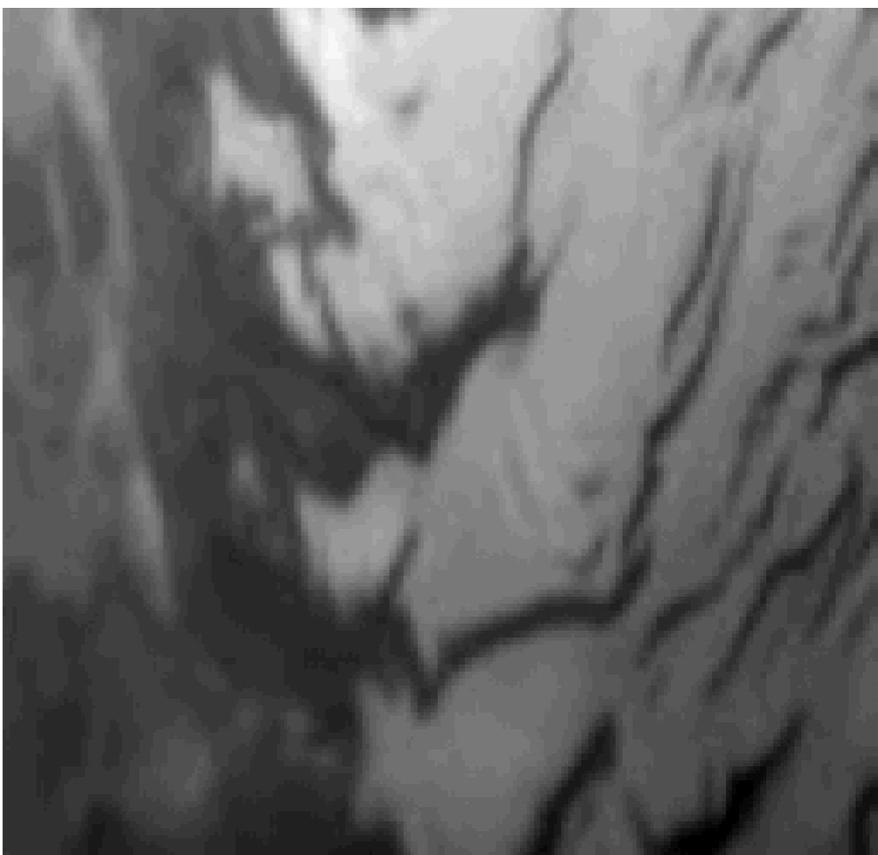
# Good performance and reasonable complexity of ITKrMM also valid for hyperspectral image inpainting



Hyperspectral data with 50 % missing pixels



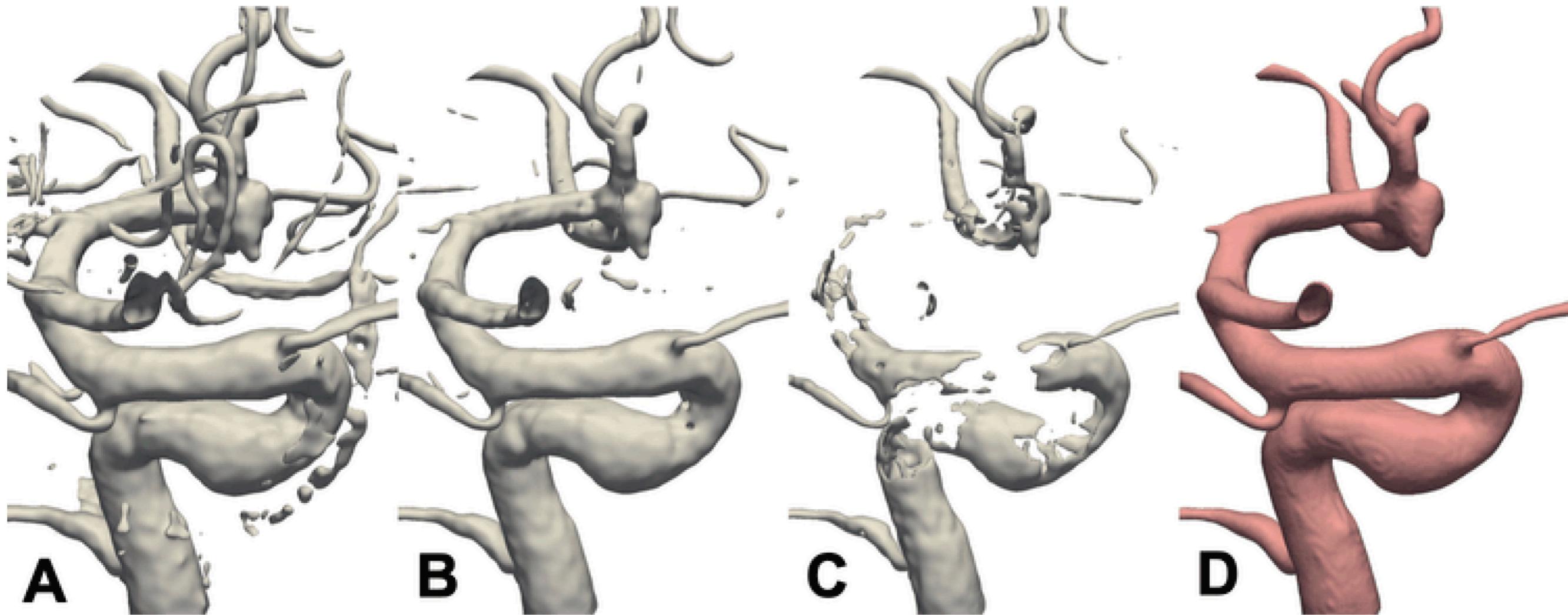
Hyperspectral data from Mars Observer, 128x128x64



Spatial recovery with the ITKrMM dictionary

Hyperspectral data available from Mars observer

# Good performance of ITKrMM for medical image denoising and segmentation



Extracted brain blood vessels from the CT image with manual segmentation (A-C) and automatic ITKrMM-based segmentation (D)

# Thanks to



**Karin Schnass,  
Uni Innsbruck**



**Jean-Luc Starck,  
CosmoStat CEA**



**Massimo Fornasier,  
TU Munich**

**Simula will contribute to the NORA activities  
with two PhD positions in AI**

